22nd International Geometry Symposium

8–10 July 2026

Süleyman Demirel University

Isparta / TÜRKİYE

Book of Abstracts

In Honor of

Prof. Dr. Mahmut ERGÜT

web page:
www.geometrysymposium.org

e-mail:
info@geometrysymposium.org

22nd INTERNATIONAL

GEOMETRY SYMPOSIUM

ABSTRACTS BOOK

Proceedings of the 22nd International Geometry Symposium

July 8-10, 2026
Isparta, Türkiye

Jointly Organized by
Süleyman Demirel
University

TABLE OF CONTENTS

Foreword **1**

Committees **2**

Invited Speakers **10**

Applications of a New Class of Derivations in Lifting Theory
Arif Salimov 11
On Strictly Locally Conformal Structures on Almost Contact Metric Manifolds
Hakan Mete Taştan 12
Reduced Biquaternions and Applications in Signal Processing
Hidayet Hüda Kösal 13
Data-driven Decision Support in Complex Situations
Stefan Wolfgang Pickl 15

Abstracts of Oral Presentations **16**

Mosaics of Some Combinatorial Designs
Kevser Soytürk, Mustafa Gezek 17
Introduction to Arakelov Geometry
Sifa Aygün, Mahmut Ergüt 18
Topological Inequalities for the Normalized Ricci–Harmonic Flow
Yasemin Soylu, Davut Karaköse 19
Laplace Operators of Some Special Parallel-Like Surfaces
Tuğba Gençer, Ali Çakmak 20
An Associated Curve via the Myller Configuration in Euclidean 3-Space
Bahar Doğan Yazıcı 21
An Analysis of Novel Associated Curves in E^n
Bedia Merih Özçetin Vatansever 22
Some Results on Three-Dimensional Trans-para-Sasakian Manifolds
Mustafa Özkan, İrem Küpeli Erken 23
η-Ricci-Bourguignon Solitons on Three-Dimensional H-paracontact Manifolds
İrem Küpeli Erken, Berna Özdamar 24
Direction Curves in Multiplicative Space
Aslı Aybüke Akdemir, İlkay Arslan Güven 25

Hasimoto Formulations of the Deformed Heisenberg Spin Equation in Minkowski Space
Ayşe Yavuz, Melek Erdoğdu 26
Statistical Structures on the Cotangent Bundle
Filiz Ocak, Tayfun Toprak, Osman Alemdar 27
Parallel Hypersurfaces and Their Applications to Rotational Hypersurfaces
Sezgin Büyükkütük, İlim Kişi, Günay Öztürk 28
Some Characterizations of Magnetic Surfaces in Euclidean Spaces in $\mathbb{R}^3$
Beyza Saruhan, Kadri Arslan 29
Helical Extension Curve in Pseudo-Galilean Space
Aykut Toplama, Mustafa Dede, Cumali Ekici 30
Geometric Characterizations of C-Loxodromes on S-Manifolds
Şaban Güvenç, Esin Akgün 31
Surfaces in Warped Product Manifold
Alev Kelleci Akbay 32
Curvature Properties of Kahler Metallic Manifolds with Constant Holomorphic Sectional Curvature
Olgun Durmaz, Aydın Gezer 33
Pythagorean Hodograph Curves in Poincare Upper Half-Plane
Aziz Yazla 34
Curvatures of Dual Curves in 2-Dimensional Setting
Nursemin Çavdar, Muhittin Evren Aydın, Mahmut Ergüt 35
Geometric Potential of Surfaces in Euclidean Spaces
Betül Bulca Sokur 36
Generalized Connections and $*$-Ricci–Bourguignon Solitons on LP-Sasakian Manifolds
Murat Altunbaş, Ayşe Karanlık Akpınar 38
Twin-Conformal Collinations
Sibel Turanlı, Aydın Gezer 39
Some Gradient Solitons on Siklos Spacetimes
Sinem Güler 40
Pedal Curves according to the Modified Orthogonal Frame in Euclidean 3-Space
Burçin Saltık Baek, Nural Yüksel, Esra Damar 41
Grey Network Games and Equal Surplus Sharing in Regional Water Transmission Under Uncertainty
Halil İbrahim Dönmez, Mehmet Onur Olgun, Sırma Zeynep Alparslan Gök 42
Some Special Collineations on an Almost Product Lorentzian Walker Three-Manifold
Büşra Aktaş, Aydın Gezer 44
On the Marginally Trapped Surfaces of a Warped Space-Time
Nurettin Cenk Turgay 45
Recent Developments of Bertrand Curves in Minkowski Space-time
Kazım İlarslan 46

Some Properties of Weak Almost Contact Manifolds
Erol Yaşar, Şemsi Eken Meriç, Erol Kılıç 47
Sequential Framed Direction Curves of Framed Curves and Their Characterizations
Medine Bakraç, Mustafa Düldül 48
A Multiplicative Calculus Perspective on AW(k)-Type Curves
Tugay Türkmen, Serpil Ünal, Semra Kaya 49
A Note on Weak Almost Contact Manifolds
Şemsi Eken Meriç, Sadık Keleş, Rıfat Güneş 50
A Smoothed Objective Penalty Function Method for Solving Constrained Optimization Problems
Nurullah Yılmaz, Dilara Kocatürk 51
A Study on Tangent Ruled Surfaces of Associated Curves in the Modified Orthogonal Frame
Nural Yüksel, Nurdan Oğraş, Süleyman Şenyurt 52
A New One-Parameter Filled Function Algorithm with Initialization Strategies for Global Optimization
Nurullah Yılmaz, Eylül Ayyıldız 53
Prolongations of Almost Hsu-Golden Structures to Tangent Bundles
Tuğba Tamirci, Mustafa Özkan 54
On Classifications of Almost Poly-Norden Pseudo-Riemannian Manifolds
Neslihan Yıldırım, Mustafa Özkan 55
On Hypersurfaces in the Minkowski Space E_1^5
Zülal Derin Yaqub, Nurettin Cenk Turgay 56
On the Main Properties of the Fiberwise Berger Type Deformed Sasaki Metric on the Coframe Bundle
Habil Fattayev 57
The Schouten-van Kampen Connection on Riemannian Concircular Structure Manifolds
Ayda Murat, Özge Özbek, Semra Zeren, Ahmet Yıldız 58
Tzitzeica Curves with Myller Configuration in 4D Euclidean Space
Zehra İşbilir, Bahar Doğan Yazıcı, Murat Tosun 60
A Study on the Relationship Between Polyhedra and Minkowski Geometries
Özcan Gelişgen, Temel Ermiş 63
Cross Product and Slant Curves in 3-Dimensional Contact Metric Spaces
Çetin Camcı 64
Hyperbolic Spinor Equations of Mannheim Curves in Minkowski 3-Space
Zeynep İsabeyoğlu, Tülay Erişir 65
On the Geometric Meaning of the Rotation Angle Around a Lightlike Axis in Minkowski-3 Space
Mehmet Duru, Harun Barış Çolakoğlu 67
New Classes of Supervectors
Dilek Demirhan, Cansel Aycan 68
A Characterization of Constant Mean Curvature Revolution Surfaces in Galilean Space
İsmet Gölgeleyen 69

Geometric Invariants and Higher-Order Variations of Paths in Lorentzian 3-Space
Fatma Almaz 70
On Non-Lightlike Framed Immersions Under Rotations and Reflections
Gülsümnur Coşkun, Gülşah Aydın Şekerci..... 71
Spherical Loxodrome and Its Applications
Melike Bozkurt, Cansel Balıkçı, Murat Babaarslan..... 72
The Characterizations of Timelike Loxodromes on Timelike Rotational Surfaces with Lightlike Axis in Minkowski 3-Space
Ferdağ Kahraman Aksoyak, Burcu Bektaş Demirci, Murat Babaarslan 73
A Class of Framed Curves whose Darboux Vector is Perpendicular to a Constant Direction
Onur Kaya 74
A Tight Frame Approach to Sparse Image Representation: Comparing Discrete Fourier and Discrete Cosine Transforms
Gülden Kapusuz……...75
On Quasi Focal Curves of Non-Null Curves with Quasi-Frame
Gamze Tarım Kandilci, Başak Yağbasan, Cumali Ekici……...76
A Study on h-Almost Yamabe Solitons Along Riemannian Maps
Mehmet Çaldır, Nergiz (Önen) Poyraz, Erol Yaşar……77
Minimisation of Curvature Energy in Parametric Curves: A Comparative Analysis of Classical and q-Gradient Descent Algorithms
Elvan Korkmaz, Serdar Soylu……79
A Direct Proof of the Steiner-Lehmus Theorem Using GeoGebra
Recep Aslaner, İrem Üstüner……80
Double-Normal Chords and Singularities in a Hyperbolic Holditch-Type Problem
Serdar Soylu……..81
Complete Biconservative Manifolds
Aykut Kayhan……...82
4x4 Matrix Representation of Hybrid Numbers, Gram Matrix and Golden Ratio
Abdullah Mağden……83
Geometric Applications in Dual Transformations
Gülsüm Yüca, Yusuf Yaylı……84
Parameter-Free Filled Functions in Global Optimization: Theory and Benchmark Testing
Gülden Kapusuz……..85
Graph-Type General Helices on a Right Circular Cylinder: Characterization Geodesicity, and Planarity
Ahmet Mollaoğulları……..86
Generalized Magnetic Curves in the 3-Dimensional Heisenberg Group
Cihan Özgür……87
On Anti-Invariant Submanifolds of LP-Sasakian Manifolds
Hülya Aytimur, Cihat Kılınç……88
Bertrand Offsets of Ruled Surfaces in 3-Dimensional Riemannian Manifolds
Kemalettin Can Deniz, Ahmet Yücesan, Gözde Özkan Tükel, Tunahan Turhan……..89
Some Results on Kahler–Norden Statistical Manifolds
Gökhan Arıkan, H. Bayram Karadağ……..91

A Study on Quaternionic Framed Curves
Ayşe Zeynep Azak, Tülay Erişir, Arzu Sürekçi, Mehmet Ali Güngör……92
Some New Characterization Results on Lightlike Submanifolds in Indefinite Contact Geometry
Burçin Doğan……...94
Slant Ruled Surfaces in 3-Dimensional Riemannian Manifolds
Hatice Sena Savcu, Tunahan Turhan, Ahmet Yücesan, Gözde Özkan Tükel……95
Mannheim Offsets of Ruled Surfaces in Simply Connected 3-Dimensional Space Forms
Yağmur Karabıyık, Gözde Özkan Tükel, Tunahan Turhan, Ahmet Yücesan…...97
Mannheim Offsets of Ruled Surfaces with B-Darboux Frame
Faik Duran, Gül Uğur Kaymanlı……98
Characterizations of Bi-Rotational Hypersurface in Euclidean 4-Space
Neriman Acar, Ferdağ Kahraman Aksoyak……..99
Pointwise Quasi Hemi-Slant Riemannian Maps into Almost Hermitian Manifolds and Casorati Inequalities
Sezin Aykurt Sepet, Meltem Karaismailoğlu, Mahmut Ergüt……100
A Homothetic Hodge–de Rham Theory and Geometric Regularization of Elliptic Boundary Value Problems
Fereidoun Sabetghadam……..101
The Algebraic Structure of $H_{(a_1,a_2,a_3)}(\varepsilon)$-Elliptic Quaternions over Zp
Hatice Çetinkaya, Gülay Koru Yücekaya……102
New Extensions on Involute–Evolute Curves in Euclidean Space and Lorentzian Space
Vildan Özdemir, Gülay Koru Yücekaya……..103
Kinematic Modeling of Spiral Staircase Climbing Motion
Bülent Karakaş, Şenay Baydaş……...105
A Study on Quaternionic and Framed Curves
Arzu Sürekçi……..106
On Tubular Surfaces in Four-Dimensional Galilean Space
Hatice Tozak, Cumali Ekici, Mustafa Dede……....108
On Almost Pseudo B-Symmetric Manifolds
Dilek Kurt, Hülya Bağdatlı Yılmaz……...109
Characterization of Plane Elastic Curves with Caputo Fractional Derivative
Bahar Abalı, Ahmet Yücesan……..110
On 3-Parameter Generalized Quaternions and Dual Quaternions with Bihyperbolic Generalized Tribonacci Number Components
Zehra İşbilir, Nurten Gürses, Murat Tosun……112
Stationary Acceleration Curves in SE(3) Under the Combescure Transformation
Kadir İçaçan, Yusuf Yaylı 117
List of Participants 118

Foreword

Dear Colleagues,

The 22nd International Geometry Symposium was held at the Süleyman Demirel University, Isparta, Turkey on 8-10 July 2026 in honor of Mahmut ERGUT. First of all, we present our deepest thanks to our supporters Süleyman Demirel University, Sakarya University, Sakarya University of Applied Science, Geometricians Society for their supports and efforts during the conference days.

Continuity in holding an international symposium is a team effort that requires dedication. For this reason, we would like to thank the symposium scientific and organizing committee members for all their support.

In addition, we would like to thank the invited speakers who participated in our symposium from Turkey and abroad, and all the participants who supported the symposium by participating in the symposium with and without papers. With the hope that the studies presented in this symposium are going to contribute to science and scientists...

On Behalf of Organizing Committee
Prof. Dr. Nihat AYYILDIZ
Süleyman Demirel University

Committees

Honorary Committee

Prof. Dr. Mehmet SALTAN	**Rector of Süleyman Demirel University**
Prof. Dr. Hamza AL	**Rector of Sakarya University**
Prof. Dr. H. Hilmi HACISALİHOĞLU	**TWMS of Honorary Chef**
Prof. Dr. Yusuf ADIGÜZEL	**Rector of Anadolu University**
Prof. Dr. Erol YAŞAR	**Rector of Mersin University**
Prof. Dr. Mehmet SARIBIYIK	**Rector of Sakarya University of Applied Sciences**

Head of Organizing Committee

Prof. Dr. Nihat AYYILDIZ	**Süleyman Demirel University**

Members of Organizing Committee

Prof. Dr. Suna SALTAN	**Süleyman Demirel University**
Prof. Dr. S. Zeynep ALPARSLAN GÖK	**Süleyman Demirel University**
Prof. Dr. Duygu Aruğaslan ÇİNÇİN	**Süleyman Demirel University**
Prof. Dr. Tunahan TURHAN	**Süleyman Demirel University**
Prof. Dr. Ahmet YÜCESAN	**Süleyman Demirel University**
Prof. Dr. Murat TOSUN	**Sakarya University**
Prof. Dr. Ayhan SARIOĞLUGİL	**Ondokuz Mayıs University**
Prof. Dr. Mahmut ERGÜT	**Tekirdağ Namık Kemal University**
Prof. Dr. Murat Kemal KARACAN	**Uşak University**
Prof. Dr. Gözde ÖZKAN TÜKEL	**Isparta University of Applied Sciences**

Prof. Dr. Mahmut AKYİĞİT	**Sakarya University**
Prof. Dr. Salim YÜCE	**Yıldız Technical University**
Prof. Dr. Bülent ALTUNKAYA	**Kırşehir Ahi Evran University**
Prof. Dr. Kazım İLARSLAN	**Kırıkkale University**
Prof. Dr. Soley ERSOY	**Sakarya University**
Prof. Dr. Osman Zeki OKUYUCU	**Bilecik Şeyh Edebali University**
Prof. Dr. Mehmet Ali GÜNGÖR	**Sakarya University**
Prof. Dr. Cengizhan MURATHAN	**Bursa Uludağ University**
Prof. Dr. Cumali EKİCİ	**Eskişehir Osmangazi University**
Prof. Dr. Rıfat GÜNEŞ	**İnönü University**
Prof. Dr. Ahmet YILDIZ	**İnönü University**
Prof. Dr. İrem KÜPELİ ERKEN	**Bursa Technical University**
Prof. Dr. Mustafa ÖZDEMİR	**Akdeniz University**
Prof. Dr. Nedim DEĞİRMENCİ	**Eskişehir Technical University**
Prof. Dr. Mustafa DÜLDÜL	**Yıldız Technical University**
Prof. Dr. Mustafa ÖZKAN	**Gazi University**
Prof. Dr. Melek ERDOĞDU	**Necmettin Erbakan University**
Prof. Dr. Abdülkadir ERDOĞAN	**Anadolu University**
Prof. Dr. Önder Gökmen YILDIZ	**Bilecik Şeyh Edebali University**
Prof. Dr. Murat BABAARSLAN	**Yozgat Bozok University**
Assoc. Prof. Dr. Nurten GÜRSES	**Yıldız Technical University**
Assoc. Prof. Dr. Ayşe YILMAZ CEYLAN	**Akdeniz University**
Assoc. Prof. Dr. Kemal EREN	**Sakarya University**

Assoc. Prof. Dr. Gülşah AYDIN ŞEKERCİ	**Süleyman Demirel University**
Assoc. Prof. Dr. Nurullah YILMAZ	**Süleyman Demirel University**
Assoc. Prof. Dr. Bahar DOĞAN YAZICI	**Bilecik Şeyh Edebali University**
Assoc. Prof. Dr. Yasemin SOYLU	**Giresun University**
Assoc. Prof. Dr. İnan ÜNAL	**Munzur University**
Assoc. Prof. Dr. Nergiz POYRAZ	**Çukurova University**
Assoc. Prof. Dr. Şemsi EKEN MERİÇ	**Mersin University**
Assist. Prof. Dr. Zekiye ÇİLOĞLU ŞAHİN	**Süleyman Demirel University**
Res. Asst. Saime KOLANCI	**Süleyman Demirel University**
Assoc. Prof. Dr. Ayşe Zeynep BİRDAL	**Sakarya University**
Assist. Prof. Dr. Mehmet Akif YETİM	**Süleyman Demirel University**
Res. Asst. Makbule KOCABIYIK ERÇOĞUL	**Süleyman Demirel University**
Res. Asst. Zehra İŞBİLİR	**Düzce University**

Members of Scientific Committee

Prof. Dr. Nihat AYYILDIZ	**Süleyman Demirel University**
Prof. Dr. Suna SALTAN	**Süleyman Demirel University**
Prof. Dr. S. Zeynep ALPARSLAN GÖK	**Süleyman Demirel University**
Prof. Dr. Duygu Aruğaslan ÇİNÇİN	**Süleyman Demirel University**
Prof. Dr. Tunahan TURHAN	**Süleyman Demirel University**
Prof. Dr. Abdullah MAGDEN	**Bursa Technical University**
Prof. Dr. Ahmet YÜCESAN	**Süleyman Demirel University**

Prof. Dr. Ali İhsan SİVRİDAĞ	**İnönü University**
Prof. Dr. Alper Osman ÖĞRENMİŞ	**Fırat University**
Prof. Dr. Arif SALIMOV	**Baku State University**
Prof. Dr. Atakan Tuğkan YAKUT	**Niğde Ömer Halisdemir University**
Prof. Dr. Ayhan TUTAR	**Ondokuz Mayıs University**
Prof. Dr. Ayşe Bayar KORKMAZOĞLU	**Eskişehir Osmangazi University**
Prof. Dr. Cengizhan MURATHAN	**Uludağ University**
Prof. Dr. Cumali EKİCİ	**Eskişehir Osmangazi University**
Prof. Dr. Erol KILIÇ	**İnönü University**
Prof. Dr. Günay ÖZTÜRK	**İzmir Demokrasi University**
Prof. Dr. H. Bayram KARADAĞ	**İnönü University**
Prof. Dr. Keziban ORBAY	**Amasya University**
Prof. Dr. Mehmet BEKTAŞ	**Fırat University**
Prof. Dr. Murat Kemal KARACAN	**Uşak University**
Prof. Dr. Mustafa DÜLDÜL	**Yıldız Technical University**
Prof. Dr. Mustafa KAZAZ	**Manisa Celal Bayar University**
Prof. Dr. Süha YILMAZ	**Dokuz Eylül University**
Prof. Dr. Süheyla EKMEKÇİ	**Eskişehir Osmangazi University**
Prof. Dr. İsmail AYDEMİR	**Ondokuz Mayıs University**
Prof. Dr. Ali ÇAKMAK	**Ağrı İbrahim Çeçen University**

Prof. Dr. Sezai KIZILTUĞ	**Erzincan Binali Yıldırım University**
Prof. Dr. Seher ASLANCI	**Alanya Alaaddin Keykubat University**
Prof. Dr. Mustafa ÖZKAN	**Gazi University**
Prof. Dr. Fatma KARAKUŞ	**Sinop University**
Prof. Dr. Bayram ŞAHİN	**Ege University**
Prof. Dr. Cihan ÖZGÜR	**İzmir Demokrasi University**
Prof. Dr. Kadri ASLAN	**Uludağ University**
Prof. Dr. Nuri KURUOĞLU	**İstanbul Gelişim University**
Prof. Dr. Yusuf YAYLI	**Ankara University**
Prof. Dr. Bülent KARAKAŞ	**Bartın University**
Prof. Dr. Halit GÜNDOĞAN	**Kırıkkale University**
Prof. Dr. Hasan Hüseyin UĞURLU	**Gazi University**
Prof. Dr. Haşim ÇAYIR	**Giresun University**
Prof. Dr. Kazım İLARSLAN	**Kırıkkale University**
Prof. Dr. Levent KULA	**Kırşehir Ahi Evran University**
Prof. Dr. Kürşat AKBULUT	**Atatürk University**
Prof. Dr. Nejmi CENGİZ	**Atatürk University**
Prof. Dr. Ahmet YILDIZ	**İnönü University**
Prof. Dr. Aydın GEZER	**Atatürk University**
Prof. Dr. Sıddıka ÖZKALDI KARAKUŞ	**Bilecik Şeyh Edebali University**

Prof. Dr. Mehmet Ali GÜNGÖR	**Sakarya University**
Prof. Dr. Murat TOSUN	**Sakarya University**
Prof. Dr. Gözde ÖZKAN TÜKEL	**Isparta University of Applied Sciences**
Prof. Dr. Soley ERSOY	**Sakarya University**
Prof. Dr. Murat BABAARSLAN	**Yozgat Bozok University**
Prof. Dr. Mustafa ÖZDEMİR	**Akdeniz University**
Prof. Dr. Nedim DEĞİRMENCİ	**Eskişehir Technical University**
Prof. Dr. Melek ERDOĞDU	**Necmettin Erbakan University**
Prof. Dr. Marian Ioan MUNTEANU	**Alexandru Ioan Cuza University**
Prof. Dr. Jemal GUVEN	**National University of Mexico**
Prof. Dr. Donghe PEI	**Northeast Normal University**
Prof. Dr. Ryszard DESZCZ	**Wroclaw University of Environmental and Life Sciences**
Prof. Dr. Luis J. ALIAS	**Universidad de Murcia**
Prof. Dr. Magdalena Daniela TODA	**University of Nis**
Prof. Dr. Ljubica VELIMIROVIC	**University of Nis**

Presentation Notice for authors and Section Chairs/Moderators

REMARKS:

1. The authors must be ready in the meeting room at least 5 minutes prior to the start of the session. Presenters must introduce themselves to the session chair(s) and upload their Oral presentations to the computer.
2. Authors must be able to present on any day of the symposium the program cannot be tailored around specific requests from individual authors to present on particular days.
3. The international research symposium program is designed for original research contributions and presentations in all research fields. Presentations scheduled in the Oral sessions are drawn from a selection of peer reviewed papers from a wide range of scientific and other disciplines of inquiry.

ROLE OF THE SESSION CHAIR/MODERATOR

The duties of the Session Chair include the following:

1. Arrive at the conference hall at least 5 minutes before the session begins. Identify the paper presenters and discussant(s) in advance and introduce yourself. Remind each presenter of the time limits that apply and describe the method you will use to alert them of time limits during the actual presentation.
2. At the start of the session, introduce yourself to the audience, announce the session/title, and offer a brief overview indicating how the papers are related.
3. Prior to each presentation, introduce the speaker, announce the paper's title, the name(s) of the author(s), and provide brief comments regarding the affiliation and/or background of each presenter. Identify the individual who will be speaking if it is someone other than the first author.
4. During the presentations enforce time limits strictly so that no author (or audience member) monopolizes someone else's time. Oral paper presentations each have 15 minutes (10 minutes for full presentation papers, 5 minutes for discussions).
5. Once presentations are complete, the remaining time can be used for informal discussion between the audience and session participants. It is your job to field questions from the audience.

Invited Speakers

Applications of a New Class of Derivations in Lifting Theory

Arif Salimov

Baku State University, Azerbaijan e-mail:asalimov@hotmail.com

Abstract

The purpose of this presentation is to investigate the liftings of $(1,1)$-tensor fields (endomorphism fields) to tensor bundles of type $(0,q)$ along pure cross-sections. In particular, most of the classical results concerning liftings in the tangent bundle are generalized within a new framework, and several formulas related to complete and horizontal lifts in the tangent bundle are unified and extended through our approach. The fundamental starting point of our study is the notion of the Φ_φ-derivation associated with a $(1,1)$-tensor field φ, acting on pure tensor fields ξ. The theory of Φ_φ-operators has been extensively developed by Tachibana [4], Yano [3].

In these works, applications of Φ_φ-operators have been primarily concerned with holomorphic objects. More precisely, when the $(1,1)$-tensor field φ determines an almost complex (or almost paracomplex) structure, a pure tensor field ξ satisfying $\Phi_\varphi\xi = 0$ is called an almost holomorphic tensor field [1, 2].

Unlike the situation in the tangent bundle, the lifting of a $(1,1)$-tensor field to a tensor bundle cannot be characterized solely by means of Lie derivatives and covariant derivatives. We show that the existence of such liftings along pure cross-sections naturally leads to a more general lifting procedure. This generalized framework is established through the introduction of generalized Φ_φ-derivations, which provide a unified treatment of various lifting constructions and reveal new geometric properties of tensor bundles.

Keywords: Lifting theory, tensor bundles, generalized derivations, pure cross-sections, Φ_φ-derivations.

2010 Mathematics Subject Classification: 53C15, 53C07, 58A32.

References

[1] A. Salimov, Applications of holomorphic functions in geometry, Springer/Birkh"auser, Singapore, 2023.

[2] A. Salimov, M. Iscan, F. Etayo, Paraholomorphic B-manifold and its properties, *Topology Appl.* **154** (2007), 925–933.

[3] K. Yano, M. Ako, On certain operators associated with tensor fields, *Kodai Math. Semin. Rep.* **20** (1968), 414–436.

[4] S. Tachibana, Analytic tensor and its generalization, *Tohoku Math. J.* **12** (1960), 208–221.

On Strictly Locally Conformal Structures on Almost Contact Metric Manifolds

Hakan Mete Taştan

Department of Mathematics, İstanbul University, İstanbul, Türkiye
e-mail:hakmete@istanbul.edu.tr

Abstract

In this talk, we investigate the existence of strictly locally conformal structures on almost contact metric manifolds. We explore the geometric properties and conditions under which these strictly (non-globally) conformal structures arise. Furthermore, to substantiate our theoretical findings, we construct some explicit illustrative examples.
Keywords: Almost contact metric manifold, locally conformal almost contact metric structures, locally transversally Kenmotsu manifolds, transversal Lee form.
2010 Mathematics Subject Classification: 53C18, 53C25.

References

[1] D. E. Blair, Contact manifolds in Riemannian geometry, Lecture Notes in Math., Springer, Berlin-Heidelberg-New York, 1976.

[2] D. Chinea, J. C. Marrero, Conformal changes of almost contact metric manifolds, *Riv. Mat. Univ. Parma* **5** (1992), 19–31.

[3] S. Dragomir, L. Ornea, Locally conformal K"ahler geometry, Birkh"auser, Boston, Basel, Berlin, 1998.

[4] A. Gray, Minimal varieties and almost Hermitian submanifolds, *Michigan Math. J.* **12** (1965), 273–287.

[5] K. Kenmotsu, A class of almost contact Riemannian manifolds, *Tohoku Math. J.* **24** (1972), 93–103.

[6] S. Sasaki, On differentiable manifolds with certain structures which are closely related to almost contact structure I, *Tohoku Math. J.* **12** (1960), 459–476.

[7] I. Vaisman, Conformal changes of almost contact metric structures, *Lect. Notes Math.* **792** (1980), 435–443.

Reduced Biquaternions and Applications in Signal Processing

Hidayet Hüda Kösal

Department of Mathematics, Sakarya University, Sakarya, Türkiye,
hhkosal@sakarya.edu.tr

Abstract

Reduced biquaternions with the parameter $p = -1$, also known as elliptic quaternions or commutative (Segre) quaternions, form a four-dimensional commutative real algebra spanned by $\{1, i, j, k\}$ with $i^2 = k^2 = -1$, $j^2 = 1$, and $ij = ji = k$. Through the idempotent units $e_1 = (1 + j)/2$ and $e_2 = (1 - j)/2$, every reduced biquaternion matrix splits into two independent complex components. In this talk, we first present Householder transformations formulated directly in the e_1–e_2 idempotent representation and, building on them, Householder-based QR and singular value decomposition (SVD) algorithms for reduced biquaternion matrices that operate in native algebraic form, avoiding representation-matrix conversion and recombination overhead. Numerical experiments confirm their accuracy and computational efficiency against real quaternion and commutative quaternion baselines. This single algebraic framework then drives three signal-processing applications where multi-channel data are encoded holistically: (i) SVD-based denoising of three-component seismic signals via Hankel embedding and low-rank truncation, (ii) SVD-based color image compression, reconstruction, and eigen-image analysis, and (iii) QR-based blind color image watermarking with key-dependent block selection and scrambling, evaluated for imperceptibility, robustness, and security.

Keywords: Reduced biquaternions, Householder transformation, QR decomposition, singular value decomposition, signal processing.
2020 Mathematics Subject Classification: 15A23, 65F25, 65F55.

References

[1] S.-C. Pei, J.-H. Chang, J.-J. Ding, M.-Y. Chen, Eigenvalues and singular value decompositions of reduced biquaternion matrices, *IEEE Trans. Circuits Syst. I: Regul. Pap.* **55** (2008), 2673–2685.

[2] H. H. Kösal, E. Kişi, M. Akyiğit, B. Çelik, Elliptic quaternion matrices: theory and algorithms, *Axioms* **13** (2024), 656.

[3] S.-C. Pei, J.-H. Chang, J.-J. Ding, Commutative reduced biquaternions and their Fourier transform for signal and image processing applications, *IEEE Trans. Signal Process.* **52** (2004), 2012–2031.

[4] Y. Li, M. Wei, F. Zhang, J. Zhao, Real structure-preserving algorithms of Householder based transformations for quaternion matrices, *J. Comput. Appl. Math.* **305** (2016), 82–91.

[5] D. Zhang, T. Jiang, G. Wang, V. I. Vasil'ev, On singular value decomposition and generalized inverse of a commutative quaternion matrix and applications, *Appl. Math. Comput.* **460** (2024), 128291.

[6] X. Zhao, B. Ye, Similarity of signal processing effect between Hankel matrix-based SVD and wavelet transform and its mechanism analysis, *Mech. Syst. Signal Process.* **23** (2009), 1062–1075.

Data-driven Decision Support in Complex Situations

- IRIS Integrated Reachback Information System as Hybrid Optimization Framework
- Digital Twins for the Anomaly Detection of Complex Supply Chains with the Support of Polytopes and Geometric Modelling
- INESS – Intelligent Network Analysis and Security Structures

Stefan Wolfgang Pickl
DKKV, COMTESSA, Munich, Germany

Abstract

The current situation faced both the decision-makers in politics in particular as well as the players in the logistics industry in generally a multitude of complex challenges. The supply chains considered currently need special attention to be optimized on complex disruptions that affect the general security of supply. As part of the research project "OPERA - Future Operations" an interdisciplinary consortium develops a trend-setting interactive visualization tool to support decision makers politics, to adapt and to explore different strategies especially with the aid of digital twins. This talk gives an overview on that project. Furthermore it presents the special sector-based approach IDEA4C as part of the IRIS project:

I. Identification of critical regions, sectors and coupling principles; **Cost–Benefit Analysis for critical sectors**.

D. Data-driven constraint optimization; **Coupled sector-based models**.

E. Exploratory strategies; Recognition of critical pathways and sectors; **Characterization of pathways**.

A. Adaptation and quantifying analysis; **Coordination of strategies**.

This 4C-approach describes a new integrated modelling suite for developing and assessing relief distribution strategies. With the aid of a digital twin "IRIS" approach, anomaly detection should be supported. This hybrid optimization framework as example for IRIS has been conceptualized and is currently being developed with and for experts in the field of disaster and emergency management, in order to tackle the real issues arising during such this crisis. INESS is based on an analytical approach which is based on polytopes.

Abstracts of Oral Presentations

Mosaics of Some Combinatorial Designs

Kevser Soytürk, Mustafa Gezek
Department of Mathematics, Tekirdağ Namık Kemal University, Tekirdağ, Türkiye,
ksoyturk@nku.edu.tr, mgezek@nku.edu.tr

Abstract

Mosaics of combinatorial designs extend classical design theory by combining several designs within a single incidence matrix. In a mosaic, the entries of the matrix are represented by different symbols or colors, where each symbol corresponds to a distinct combinatorial design with the same set of points and blocks. Thus, the incidence structure is partitioned into multiple designs that coexist within one matrix framework. This approach provides a new perspective for constructing and studying combinatorial designs and allows different designs to be analyzed simultaneously within a unified structure. In this talk, we review several well-known constructions of mosaics of combinatorial designs and discuss some of the main open problems. We also present our recent approach and calculations addressing these problems, together with the results obtained in this direction.

Keywords: Combinatorial Design, Mosaics of Designs, Incidence Matrix.
2010 Mathematics Subject Classification: 05B05, 05B20.

References

[1] O. W. Gnilke, M. Greferath, M. O. Pavčević, Mosaics of combinatorial designs, *Designs, Codes and Cryptography* 86 (2018), 85–95.

[2] V. Krčadinac, Small examples of mosaics of combinatorial designs, *Examples and Counterexamples* 6 (2024), 100163.

Introduction to Arakelov Geometry

Şifa Aygün, Mahmut Ergüt
Department of Mathematics, Tekirdağ Namık Kemal University, Tekirdağ, Türkiye,
1258124106@nku.edu.tr, mergut@nku.edu.tr

Abstract

Arakelov geometry is one of the central subjects of modern arithmetic geometry, combining algebraic geometry, number theory, and complex differential geometry within a unified framework. This talk presents an introductory overview of the fundamental ideas of Arakelov geometry and the analogy between number fields and function fields that motivates the theory.

The presentation begins with the geometric interpretation of valuations and the role of Archimedean places in arithmetic geometry. Height functions, Weil functions, and Néron functions are introduced in order to explain how arithmetic complexity can be measured geometrically. The theory of metrized line bundles and Hermitian vector bundles is then discussed, emphasizing the interaction between algebraic and analytic structures.

Furthermore, Chern forms and positivity are considered from the perspective of complex differential geometry, leading to the construction of Arakelov divisors and arithmetic intersection theory. Finally, several important applications of Arakelov geometry are mentioned, including height theory, Faltings' theorem, and the study of rational points.

Keywords: Arakelov geometry, arithmetic geometry, Hermitian vector bundles, height functions, arithmetic intersection theory.

2010 Mathematics Subject Classification: 14G40.

References

[1] S. Lang, *Introduction to Arakelov Theory*, Springer-Verlag, New York, 1988.

Topological Inequalities for the Normalized Ricci–Harmonic Flow

Yasemin Soylu, Davut Karaköse
Department of Mathematics, Giresun University, Giresun, Türkiye,
yasemin.soylu@giresun.edu.tr
Department of Mathematics, Giresun University, Giresun, Türkiye,
davut27200@gmail.com

Abstract

The Ricci–harmonic flow, introduced by List and further developed by Müller, provides a natural coupling between the Ricci flow and the harmonic map flow. In this work, we investigate the topological consequences of the long-time behavior of the normalized Ricci–harmonic flow on closed four-dimensional manifolds.

In particular, we study Euler characteristic and Hitchin–Thorpe-type inequalities in the setting of the normalized Ricci–harmonic flow. To this end, we derive evolution equations for the modified scalar curvature and its average, establish several integral identities, and obtain a Gauss–Bonnet-type formula expressed in terms of the modified Ricci tensor. The resulting formulas incorporate additional contributions arising from the harmonic map component and provide a natural extension of corresponding results known for the classical Ricci flow.

Keywords: Normalized Ricci–harmonic flow, Euler characteristic, Hitchin–Thorpe inequality, Gauss–Bonnet formula.
2010 Mathematics Subject Classification: 53C44, 53C21.

References

[1] M. Limoncu,The Euler Characteristic and Signature of Four-Dimensional Closed Manifolds and the Normalized Ricci Flow Equation, *Geom. Dedicata* **180** (2016), 229–239.

[2] B. List, Evolution of an Extended Ricci Flow System, *Commun. Anal. Geom.* **16** (2008), 1007–1048.

[3] R. Müller, Ricci Flow Coupled with Harmonic Map Flow, *Ann. Sci. Éc. Norm. Supér.* **45** (2012), 101–142.

[4] Y. Li, Long Time Existence and Bounded Scalar Curvature in the Ricci–Harmonic Flow, *J. Differential Equations* **265** (2018), 69–97.

[5] Y. Li, Local Curvature Estimates for the Ricci–Harmonic Flow, *Nonlinear Analysis* **222** (2022), 112961.

[6] G. Wu and Y. Zheng, On the Extension of Ricci Harmonic Flow, *Results in Mathematics* **75** (2020), 55.

Laplace Operators of Some Special Parallel-Like Surfaces

Tuğba Gençer, Ali Çakmak

Department of Mathematics, Faculty of Arts and Sciences, Ağrı İbrahim Çeçen Unıversity, Ağrı, Turkey, acakmak@agri.edu.tr
Department of Mathematics, Garduate Education Institute, Ağrı İbrahim Çeçen Unıversity, Ağrı, Turkey, tubagncr@gmail.com

Abstract

This study aims to examine some parallel like surfaces, a special case of surfaces which is an important topic in differential geometry, from the perspective of the Laplacian operator. Using the parametric expressions of some special parallel-like surfaces, the Laplacian operator is calculated from the metric coefficients according to the first fundamental form. Furthermore, the relationship between the Laplacian and harmonic structure is examined, and the harmonicity of some special parallel-like surfaces is investigated under specific conditions. Especially, visualizations of various surfaces are created using the Python programming language to support theoretical calculations. This allowed the geometric properties of the surfaces to be examined graphically, and the results are interpreted.

Keywords: parallel surfaces, some parallel-like surfaces, Laplace operators, harmonic functions.
2010 Mathematics Subject Classification: 53A02, 53A05.

References

[1] Ö. Biçgin, Laplace operator with respect to the fundamental form in some surfaces in Galilean space, M.Sc. Thesis, Balıkesir University Institute of Science, Balıkesir, 2020.

[2] A. Çakmak, Some special surfaces at a constant distance from the edge of regression, Ph.D. Thesis, Atatürk University Graduate School of Natural and Applied Sciences Department of Mathematics, Erzurum, 2015.

[3] A. Çakmak, Ö. Tarakcı, The image curves on regression on a surface of revolution, *Appl. Math. Sci.* **27** (2016), 74–85.

[4] F. Diyab, B. Reddy, Comparison of Laplace–Beltrami operator eigenvalues on Riemannian manifolds, *Eur. J. Math. Stat.* **3** (2022), 56–61.

[5] G. Karaaslan Yıldız, Translation surfaces, M.Sc. Thesis, Erciyes University Graduate School of Natural and Applied Sciences, Kayseri, 2018.

[6] B. Senoussi, M. Bekkar, Helicoidal surfaces with $\Delta_r^J = Ar$ in 3-dimensional Euclidean space, *Tsukuba J. Math.* **37** (2015), 437–448.

[7] Ö. Tarakcı, Hypersurfaces at a constant distance from the edge of regressions on another hypersurfaces, Ph.D. Thesis, Ankara University Graduate School of Natural and Applied Sciences Department of Mathematics, Ankara, 2002.

An Associated Curve via the Myller Configuration in Euclidean 3-Space

Bahar Doğan Yazıcı

Department of Mathematics, Bilecik Şeyh Edebali University, Bilecik, Türkiye,
bahar.dogan@bilecik.edu.tr

Abstract

In this study, we investigate an associated curve via the Myller configuration in Euclidean 3-space. We establish the relations between the moving frames and invariants of the curve and characterize its geometric properties in terms of the Myller invariants. In particular, we derive the relationships between its curvatures and examine the conditions under which the curve forms Bertrand and Mannheim curve pairs. These results reveal the geometric interaction between the associated curve and the structure of the Myller configuration.

Keywords: Myller configuration, integral curve, Bertrand curves, Mannheim curves.
2010 Mathematics Subject Classification: 53A04 53A17.

References

[1] M. P. do Carmo, Differential geometry of curves and surfaces, *Prentice-Hall*, Saddle River, 1976.

[2] F. Frenet, Sur les courbes à double courbure, *J. Math. Pures Appl.* **17** (1852), 437–447.

[3] R. Miron, The geometry of Myller configurations, applications to theory of surfaces and nonholonomic manifolds, *Romanian Acad.*, 2010.

[4] G. Macsim, A. Mihai, A. Olteanu, On rectifying-type curves in a Myller configuration, *Bull. Korean Math. Soc.* **56** (2019), 383–390.

[5] Z.İşbilir, B. Doğan Yazıcı, M. Tosun, On construction of Mannheim curves with Myller configuration, *Turkish J. Math.* (2026), in press.

[6] J. Choi, Y. Kim, Associated curves of a Frenet curve and their applications, *Appl. Math. Comput.* **218** (2012), 9116–9124.

[7] S. K. Nurkan, İ. A. Güven, M. K. Karacan, Characterizations of adjoint curves in Euclidean 3-space, *Proc. Natl. Acad. Sci. India Sect. A Phys. Sci.* **89** (2019), 155–161.

[8] A. Elsharkawy, H. Baizeed, Some integral curves according to quasi-frame in Euclidean 3-space, *Sci. Afr.* **27** (2025).

[9] T. Korpinar, M. T. Sarıaydın, E. Turhan, Associated curves relative to the Bishop frame in Euclidean 3-space, *Adv. Model. Optim.* **15** (2013), 713–717.

[10] N. Macit, M. Düldül, New associated curves of a Frenet curve in E^3 and E^4, *Turkish J. Math.* **38** (2014), 1023–1037.

An Analysis of Novel Associated Curves in $\mathbb{E}^n$

Bedia Merih Özçetin Vatansever

Department of Avionics, Faculty of Applied Sciences, Istanbul Gelisim University, Istanbul, Türkiye, bmozcetin@gelisim.edu.tr

Abstract

We define three new types of associated curves of a Frenet curve in Euclidean n-space and investigate the relationships between the Frenet apparatus of the original curve and those of the associated curves.

Keywords: Integral curve, associated curve, slant helix.
2010 Mathematics Subject Classification: 53A04.

References

[1] H. H. Hacısalihoğlu, Differential Geometry, Malatya, Türkiye: Inonu University Faculty of Arts and Sciences Publications, 1980.

[2] H. Gluck, Higher curvatures of curves in Euclidean space, *Amer. Math. Monthly* **73(7)** (1966), 699-704.

[3] I. Gök, Ç. Camcı, H. H. Hacısalihoğlu, V_n-slant helices in Euclidean n-space $\mathbb{E}^n$, *Math. Commun.* **14(2)** (2009), 317-329.

[4] J. H. Choi, Y. H. Kim, Associated curves of a Frenet curve and their applications, *Appl. Math. Comput.* **218(18)** (2012), 9116–9124.

[5] L. S. Moreira, Geometric analogy and products of vectors in n dimensions, *Adv. Linear Alg. Matrix Theory* **3(1)** (2013), 1–6.

[6] N. Macit, M. Düldül, Some new associated curves of a Frenet curve in $\mathbb{E}^3$ and $\mathbb{E}^4$, *Turk. J. Math.* **38(6)** (2014), 1023-1037.

Some Results on Three-Dimensional Trans-para-Sasakian Manifolds

Mustafa Özkan, İrem Küpeli Erken

Department of Mathematics, Bursa Technical University, Bursa, Türkiye, mustafa.ozkan@btu.edu.tr

Department of Mathematics, Bursa Technical University, Bursa, Türkiye, irem.erken@btu.edu.tr

Abstract

Trans-para-Sasakian manifolds, introduced by Zamkovoy [4], play a significant role in the field of paracontact geometry and bring together a broad class of manifolds under a single framework. If a paracontact manifold $(M^{2n+1}, \phi, \eta, \xi, g)$ satisfies

$$(\nabla_X \phi)Y = \alpha[-g(X,Y)\xi + \eta(Y)X] + \beta[g(X,\phi Y)\xi + \eta(Y)\phi X],$$

then the manifold is said to be a *trans-para-Sasakian* manifold. Here, α and β are smooth functions on the manifold. Investigations on these structures have gained interest and their general curvature properties have been examined [1, 2, 3].

In this talk, some results on a three-dimensional trans-para-Sasakian manifolds will be presented and new examples will be introduced.

Keywords: Trans-para-Sasakian manifolds, 3-dimensional trans-para-Sasakian manifold.
2010 Mathematics Subject Classification: 53B30, 53C25.

References

[1] İ. Küpeli Erken, M. Özkan, A study on trans-para-Sasakian manifolds, *J. Innov. Sci. Eng.* **8** (2024), 226–232.

[2] M. Özkan, İ. Küpeli Erken, Trans-para-Sasakian manifolds satisfying certain curvature conditions, *Khayyam J. Math.* **10** (2024), 394–402.

[3] M. Özkan, İ. Küpeli Erken, U. C. De, On trans-para-Sasakian manifolds, *Filomat* **38** (2024), 10641–10652.

[4] S. Zamkovoy, On the geometry of trans-para-Sasakian manifolds, *Filomat* **33** (2019), 6015–6024.

η-Ricci-Bourguignon Solitons on Three-Dimensional H-paracontact Manifolds

İrem Küpeli Erken, Berna Özdamar
Department of Mathematics, Bursa Technical University, Bursa, Türkiye,
irem.erken@btu.edu.tr
Institute of Science, Bursa Technical University, Bursa, Türkiye,
bernaozdamar2000@gmail.com

Abstract

This works explores the geometric properties of η-Ricci-Bourguignon solitons and their gradient counterparts within the framework of 3-dimensional H-paracontact manifolds. Our main focus lies in characterizing these solitons through the structural behavior of the associated operator h. To complement our theoretical results, we also construct examples that demonstrate the validity and applicability of our results [1, 2, 3, 4].

Keywords: Paracontact Metric Manifolds, Paracontact (κ, μ)-Manifolds, η-Ricci-Bourguignon Solitons, gradient η-Ricci-Bourguignon Solitons.
2010 Mathematics Subject Classification: 53B30, 53C25.

References

[1] İ. Küpeli Erken, C. Murathan, A study of three-dimensional paracontact (κ, μ, ν)-spaces, *Int. J. Geom. Methods Mod. Phys.* **14** (2017).

[2] İ. Küpeli Erken, S. Nur Emetli, η-Ricci-Bourguignon solitons on K-paracontact and paracontact metric $(\kappa \neq -1, \mu)$-manifolds, *Int. J. Maps Math.* **8** (2025), 622–641.

[3] T. Mandal, U. C. De, A. Sarkar, h-Ricci-Bourguignon solitons on three-dimensional (almost) coKahler manifolds, *Math. Methods Appl. Sci.* **10** (2024), 1–14.

[4] S. Zamkovoy, Canonical connections on paracontact manifolds, *Ann. Glob. Anal. Geom.* **36** (2009), 37–60.

Direction Curves in Multiplicative Space

Aslı Aybüke Akdemir, İlkay Arslan Güven

Department of Mathematics, Gaziantep University, Gaziantep, Turkey,
akdemirasliaybuke@gmail.com
Department of Mathematics, Gaziantep University, Gaziantep, Turkey,
iarslan@gantep.edu.tr

Abstract

The purpose of this paper is to investigate and characterize several classes of curves in multiplicative (non-Newtonian) geometry. Within the framework of non-Newtonian calculus, we develop the theory of associated curves by introducing multiplicative Frenet frames, multiplicative curvature, and multiplicative torsion. Using these notions, a systematic approach is established for defining and studying multiplicative integral curves associated with vector fields. Particular attention is devoted to multiplicative principal direction curves, multiplicative donor curves, and their higher-order generalizations. The characterization of multiplicative direction curves is obtained through the expression of the position vector with respect to the multiplicative Frenet frame. Furthermore, several equivalence theorems relating multiplicative general helices, multiplicative planar curves, and donor curve constructions are presented. Illustrative examples are included to demonstrate the construction of complex multiplicative curves from simpler geometric objects. The results contribute to the literature on non-Newtonian differential geometry and provide a theoretical basis for further applications in mathematical modeling and theoretical physics.

Keywords: Multiplicative space, multiplicative principal-direction curve, multiplicative principal-donor curve.
2010 Mathematics Subject Classification: 53A04, 53A17.

References

[1] S. G. Georgiev, Multiplicative differential geometry, CRC Press, Taylor and Francis Group, Boca Raton, 2022.

[2] A. Has, B. Yılmaz, A non-Newtonian some partner curves in multiplicative Euclidean space E_*^3, *Int. Electron. J. Geom.* **18** (2025), 97–110.

[3] A. Has, B. Yılmaz, On non-Newtonian helices in multiplicative Euclidean space E_*^3, *Fundam. Contemp. Math. Sci.* **6** (2025), 196–217.

[4] J. Choi, Y. Kim, Associated curves of a Frenet curve and their applications, *Appl. Math. Comput.* **218** (2012), 9116–9124.

Hasimoto Formulations of the Deformed Heisenberg Spin Equation in Minkowski Space

Ayşe Yavuz, Melek Erdoğdu

Department of Mathematics and Science Education, Necmettin Erbakan University, Konya, Türkiye, ayasar@erbakan.edu.tr

Department of Mathematics and Computer Sciences, Necmettin Erbakan University, Konya, Türkiye, merdogdu@erbakan.edu.tr

Abstract

This study investigates Deformed Heisenberg Spin Equation (DHSE) in three-dimensional Minkowski space using complex and hyperbolic vector formulations. The geometric properties of timelike curves and the related Hasimoto functions are examined within both approaches. The findings show that in addition to DHSE'S integrable structure, complex and hyperbolic formulations result in various differential relations.

Keywords: Deformed Heisenberg Spin Equation, Timelike Curves, Minkowski Space.
2010 Mathematics Subject Classification: 53A05, 53A35.

References

[1] C. Rogers, W. K. Schief, Backlund and Darboux transformations, geometry of modern applications in soliton theory, Cambridge University Press, 2002.

[2] M. J. Ablowitz, H. Segur, Solitons and the inverse scattering transform, SIAM, Philadelphia, PA, 1981.

[3] G. L. Lamb Jr., Solitons on moving space curves, *J. Math. Phys.* **18** (1977), 1654–1661.

[4] H. Hasimoto, A soliton on a vortex filament, *J. Fluid Mech.* **51** (1972), 477–485.

[5] I. Klapper, M. Tabor, A new twist in the kinematics and elastic dynamics of thin filaments and ribbons, *J. Phys. A* **27** (1994), 4919.

[6] A. Yavuz, M. Erdoğan, A geometric approach to timelike flows in terms of anholonomic coordinates, *Honam Math. J.* **44** (2022), 259–270.

[7] A. W. Marris, S. L. Passman, Vector fields and flows on developable surfaces, *Arch. Ration. Mech. Anal.* **32** (1969), 29–86.

[8] D. W. Yoon, Z. K. Yüzbaşı, Integrable geometric evolution equations through a deformed Heisenberg spin equation, *J. Geom. Phys.* **214** (2025), 105534.

[9] M. Bektaş, D. W. Yoon, Z. K. Yüzbaşı, Geometric methodology for analyzing timelike curve flows in Minkowski space, *Results Math.* **79** (2024), 154.

Statistical Structures on the Cotangent Bundle

Filiz Ocak, Tayfun Toprak, Osman Alemdar
Department of Mathematics, Karadeniz Technical University, Trabzon, TÜRKİYE,
filiz.ocak@ktu.edu.tr
tayfuntoprak2525@gmail.com
osmannalemdar@gmail.com

Abstract

A statistical manifold is a triple (M, h, ∇), where M is a differentiable manifold endowed with a statistical structure (h, ∇) consisting of a Riemannian metric h and an affine connection ∇. Moreover, the covariant derivative of the metric tensor is assumed to be symmetric. In this study, we determine the necessary and sufficient conditions under which the cotangent bundle equipped with the Sasaki metric and a linear connection $\bar{\nabla}$ becomes a statistical manifold.

Keywords: Cotangent bundle, statistical manifold.
2010 Mathematics Subject Classification: 53B05, 53C25.

Acknowledgement: This study was supported by Scientific and Technological Research Council of Türkiye (TUBITAK) under the Grant Number 125F519. The authors thank to TUBITAK for their supports.

References

[1] V. Balan, E. Peyghan, E. Sharahi, Statistical structures on the tangent bundle of a statistical manifold with Sasaki metric, *Hacet. J. Math. Stat.* **49** (2020), 120–135.

[2] F. Ocak, Notes on the Sasaki metrics in cotangent bundle, *C. R. Acad. Bulg. Sci.* **72** (2019), 871–879.

[3] A.A. Salimov, F. Agca, Some properties of Sasakian metrics in cotangent bundles, *Mediterr. J. Math.* **8**(2011), 243–255.

[4] K. Yano, S. Ishihara, Tangent and Cotangent Bundles, Mercel Dekker. Inc., New York, 1973.

Parallel Hypersurfaces and Their Applications to Rotational Hypersurfaces

Sezgin Büyükkütük, İlim Kişi, Günay Öztürk
Department of Mechanics, Kocaeli University, Kocaeli, Türkiye,
sezgin.buyukkutuk@kocaeli.edu.tr
Department of Mathematics, Kocaeli University, Kocaeli, Türkiye,
ilim.ayvaz@kocaeli.edu.tr
Department of Mathematics, İzmir Democracy University, Izmir, Türkiye,
gunay.ozturk@idu.edu.tr

Abstract

This paper investigates parallel hypersurfaces in four-dimensional Euclidean space $\mathbb{E}^4$ and establishes explicit formulas for their Gaussian and mean curvatures in terms of the curvature functions of the original hypersurface. Conditions characterizing flat and minimal parallel hypersurfaces are derived and analyzed. The theoretical framework is further illustrated through several important classes of hypersurfaces, including rotational hypersurfaces, hyperspheres, catenoidal hypersurfaces, and helicoidal hypersurfaces. For each case, detailed curvature analyzes are presented together with graphical visualizations. The obtained results generalize classical curvature relationships from three-dimensional geometry to higher-dimensional settings and contribute to a deeper understanding of the geometric behavior of parallel hypersurfaces. Moreover, these findings have potential applications in differential geometry, geometric modeling, and computational design.

Keywords: Catenoid hypersurface, Helicoidal Hypersurface, Parallel hypersurface, Rotational hypersurface.
2010 Mathematics Subject Classification: 53A07; 53C42; 53B25.

References

[1] C. Dupin, Applications de geometrie et de mecanique, Bachelier, Paris, 1822.

[2] M. P. do Carmo, Differential geometry of curves and surfaces, Prentice Hall, New Jersey, 1976.

[3] B. Y. Chen, J. Veken, Complete classification of parallel surfaces in 4-dimensional Lorentzian space forms, *Tohoku Math. J.* **61** (2009), 1–40.

[4] H. Pottmann, S. Leopoldseder, A concept for parametric surface fitting which avoids the parametrization problem, *Comput. Aided Geom. Design* **20** (2003), 343–362.

[5] B. O'Neill, Elementary differential geometry, Academic Press, USA, 2006.

[6] B. Y. Chen, Geometry of submanifolds, M. Dekker, New York, 1973.

Some Characterizations of Magnetic Surfaces in Euclidean Spaces in $\mathbb{R}^3$

Beyza Saruhan, Kadri Arslan

Department of Mathematics, Bursa Uludağ University, Bursa, Türkiye,
16beyzasaruhan@gmail.com arslan@uludag.edu.tr

Abstract

Analytical surfaces have an important place in differential geometry. These surfaces are frequently used in geometric design. The ability to define such surfaces with parametric and implicit equations is very useful for modeling with "computer-aided geometric design". In this study, we consider some characterizations of magnetic surfaces, which are an important type of analytical surfaces. Carved surfaces and also Monge surfaces are interesting examples of magnetic surfaces. Among them, molding surfaces are quite impressive in terms of aesthetics in architectural design and exterior cladding of buildings.

References

[1] R. Dandoloff, R. Balakrishnan, Quantum effective potential, electron transport and conformons in biopolymers, *J. Phys. A* **38** (2005), 6121.

[2] J. Filipova, M. Rynkovskaya, Carved Monge surfaces as new forms in the architecture, *MATEC Web Conf.* **95** (2017).

[3] M. Gil-oulbé, A. J. I. Ndomilep, Geometry and classification of carved Monge surfaces, *J. Phys. Conf. Ser.* **1687** (2020), 1–10.

[4] A. Gray, Modern differential geometry of curves and surfaces, CRC Press Inc., USA, 1993.

[5] S. N. Krivoshapko, V. N. Ivanov, Encyclopedia of analytical surfaces, Springer International Publishing, Switzerland, 2015.

[6] Z. Özdemir, I. Gök, F. N. Ekmekci, A geometric view of magnetic surfaces and magnetic curves, *Int. Electron. J. Geom.* **12** (2019), 126–134.

[7] T. S. Pedersen, A. H. Boozer, Confinement of nonneutral plasmas on magnetic surfaces, *Phys. Rev. Lett.* **88** (2002), 205002.

[8] B. Saruhan, K. Arslan, B. B. Sokur, Monge surfaces with their applications, In: Turan-Fundamental Science Symposium, Yıldız Technical University, Istanbul, June 23–25, 2025.

Helical Extension Curve in Pseudo-Galilean Space

Aykut Toplama, Mustafa Dede, Cumali Ekici

Department of Mathematics, University of Kilis 7 Aralık, Kilis, Türkiye, aykut.toplama@kilis.edu.tr

Department of Mathematics, University of Kilis 7 Aralık, Kilis, Türkiye, mustafadede@kilis.edu.tr

Department of Mathematics and Computer Science, University of Eskişehir Osmangazi, Eskişehir, Türkiye, cekici@ogu.edu.tr

Abstract

In this paper, the helical expansion curve defined as a new type of curve in Pseudo-Galilean space, is examined in detail, and fundamental characterizations of this specific curve are constructed. In addition, an example is provided to illustrate the structure of this specific curve type, and various graphical representations are produced.

Keywords: Helical Extension curve, Frenet Frame, Pseudo-Galilean Space.
2010 Mathematics Subject Classification: 53A35.

References

[1] O. R"oschel, Die geometrie des Galileischen raumes, Habilitationsschrift, Leoben, 1984.

[2] I. M. Yaglom, A simple non-Euclidean geometry and its physical basis, Springer-Verlag, New York, 2012.

[3] B. O'Neill, Elementary differential geometry, Academic Press, Elsevier, 2006.

[4] B. Divjak, Curves in pseudo-Galilean geometry, *Ann. Univ. Sci. Budapest. Rolando Eötvös Nom. Sect. Math.* **41** (1998), 117–128.

[5] M. Dede, Helical extension curve of a space curve, *Mediterr. J. Math.* **18** (2021), 6.

Geometric Characterizations of C-Loxodromes on S-Manifolds

Şaban Güvenç, Esin Akgün
Department of Mathematics, Balıkesir University, Balıkesir, Türkiye,
sguvenc@balikesir.edu.tr, es.nakgun@gmail.com

Abstract

In this study, we investigate the geometric properties of C-loxodromes on S-manifolds. An S-manifold is equipped with a metric, a tensor field f, and a set of s characteristic vector fields ξ_i along with their dual 1-forms η_i. A C-loxodrome is explicitly defined by the equation $\nabla_T T = r\big(\sum_{i=1}^{s} \eta_i(T)\big) fT$, where r is a structural constant. Based on this governing equation, we prove that the contact angles are conserved; thus, every C-loxodrome is inherently a θ_i-slant curve satisfying the condition $\eta_i(T) = \cos\theta_i = constant$.

From this perspective, we establish that C-loxodromes inherently act as a subclass of magnetic curves. By utilizing the relation $q = -r\sum_{i=1}^{s} \eta_i(T)$, the C-loxodrome equation perfectly reduces to the classical magnetic curve equation $\nabla_T T = -qfT$ with a constant magnetic charge q. This equivalence reveals that Legendre C-loxodromes ($\eta_i(T) = 0$) are entirely free from magnetic effects, thus naturally behaving as geodesics. Furthermore, by utilizing a coordinate-free projection method, we conclude that even non-Legendre curves can form geodesic trajectories provided that the sum of the contact angles neutralizes ($\sum \cos\theta_i = 0$). This geometric neutralization highlights a unique force-damping mechanism inherent to the multiple characteristic structure of S-manifolds.

Keywords: S-manifold, C-loxodrome, magnetic curve, slant curve, geodesic.
2020 Mathematics Subject Classification: 53C25, 53C22, 53B20.

References

[1] E. Akgün, C-Loxodromes on S-manifolds and their geometric properties, Master's Thesis, Balıkesir University, (in progress).

[2] J.-E. Lee, Loxodromes and transformations in pseudo-Hermitian geometry, *Commun. Korean Math. Soc.* **36** (2021), 817–827.

[3] Ş. Güvenç, C. Özgür, On slant magnetic curves in S-manifolds, *J. Nonlinear Math. Phys.* **26** (2019), 536–554.

[4] Ş. Güvenç, C. Özgür, Magnetic curves in homothetic s-th Sasakian manifolds, *Mathematics* **13** (2025), 159.

[5] Ş. Güvenç, An extended family of slant curves in S-manifolds, *Math. Sci. Appl. E-Notes* **8** (2020), 69–77.

Surfaces in Warped Product Manifold

Alev Kelleci Akbay

Department of Basic Engineering Sciences, Iskenderun Technical University, Hatay, Turkey, alev.kelleciakbay@iste.edu.tr

Abstract

Let $f: I \subseteq \mathbb{R} \to \mathbb{R}$ be a smooth strictly positive function on an open interval I and consider the warped product of I and the Minkowski 2-plane $M = \mathbb{E}_1^2$ with warping function f:

$$(\widetilde{M}, \widetilde{g}) = I \times_f \mathbb{E}_1^2 = \left(I \times M, dt^2 + f(t)^2 \left(dx^2 - dy^2\right)\right)$$

where t is the coordinate on I and x and y are coordinates on M.

Studies of surfaces in warped product manifolds $I \times M$ have been investigated by several researchers, see [1], [2], [3], [4], [6], [7], and the references therein. In [4], Dursun obtained a classifcation of all space-like and time-like constant angle surfaces in $I \times_f \mathbb{E}_1^2$.

In this study, we will focus on how to obtain the curvatures of surface M in $\widetilde{M} = I \times_f \mathbb{E}_1^2$.

Keywords: Warped product manifold, intrinsic curvature, extrinsic curvature.
2010 Mathematics Subject Classification: 53B25, 53C40.

References

[1] J. A. Aledo, R. M. Rubio, Stable minimal surfaces in Riemannian warped products, *J. Geom. Anal.* **27** (2017), 65–78.

[2] L. J. Alias, M. Dajczer, Constant mean curvature graphs in a class of warped product spaces, *Geom. Dedicata* **131** (2008), 173–179.

[3] F. Dillen, M. I. Munteanu, J. Van der Veken, L. Vrancken, Classification of constant angle surfaces in a warped product, *Balkan J. Geom. Appl.* **16** (2011), 35–47.

[4] U. Dursun, Constant angle surfaces in the Lorentzian warped product manifold $I \times_f \mathbb{E}_1^2$, *Turkish J. Math.* **46** (2022).

[5] R. L. Bishop, B. O'Neill, Manifolds of negative curvature, *Trans. Amer. Math. Soc.* **145** (1969), 1–49.

[6] B. Y. Chen, Differential geometry of warped product manifolds and submanifolds, World Scientific Publishing, 2017.

[7] B. Y. Chen, Geometry of warped product submanifolds: a survey, *J. Adv. Math. Stud.* **6** (2013), 1–43.

[8] B. O'Neill, Semi-Riemannian geometry with applications to relativity, Pure and Applied Mathematics, Vol. 103, Academic Press, New York, 1983.

Curvature Properties of Kähler Metallic Manifolds with Constant Holomorphic Sectional Curvature

Olgun Durmaz, Aydın Gezer
Department of Mathematics, Erzurum Technical University, Erzurum, Türkiye,
olgun.durmaz@erzurum.edu.tr
Department of Mathematics, Atatürk University, Erzurum, Türkiye,
aydingzr@gmail.com

Abstract

In this paper, we introduce the notion of Kähler metallic manifolds and investigate their fundamental curvature properties. By establishing a correspondence between complex metallic structures and classical complex structures, we show that a Kähler metallic manifold is equivalent to a classical Kähler manifold and derive explicit relations between their associated geometric tensors. In particular, we prove that the J_M-holomorphic sectional curvature coincides with the classical holomorphic sectional curvature. We then study Kähler metallic manifolds with constant J_M-holomorphic sectional curvature and obtain explicit formulas relating sectional curvature, J_M-holomorphic sectional curvature, and J_M-holomorphic bisectional curvature. As a consequence, we characterize complex-metallic space forms and establish several curvature identities analogous to those of classical complex space forms. Finally, we prove that every Kähler metallic manifold with constant J_M-holomorphic sectional curvature is Einstein and determine its Ricci tensor explicitly. These results show that the curvature theory of Kähler metallic manifolds naturally extends the classical theory of Kähler manifolds while preserving their essential geometric properties.

Keywords: Complex-metallic structure, Kähler metallic manifold, Holomorphic sectional curvature.
2010 Mathematics Subject Classification: 53B35, 57R15.

References

[1] C. E. Hretcanu, V. Şutu, On the geometry of the K"ahler golden manifold, *Axioms* **14** (2025), 564–580.

[2] H. Bouzir, G. Beldjilali, Almost hermitian golden manifolds, *Balkan J. Geom. Appl.* **26** (2021), 23–32.

[3] S. Turanlı, A. Gezer, H. Çakıcıoğlu, Metallic K"ahler and nearly metallic K"ahler manifolds, *Int. J. Geom. Methods Mod. Phys.* **18** (2021).

[4] L. Vanhecke, Some almost hermitian manifolds with constant holomorphic sectional curvature, *J. Differential Geom.* **12** (1977), 461–471.

[5] V. W. Spinadel, The metallic means family and multifractal spectra, *Nonlinear Anal. Ser. B: Real World Appl.* **36** (1999), 721–745.

Pythagorean Hodograph Curves in Poincaré Upper Half-Plane

Aziz Yazla

Department of Mathematics, Selçuk University, Konya, Türkiye, azizyazla@gmail.com

Abstract

In this talk, we define a class of rational hyperbolic Pythagorean hodograph curves in Poincaré upper half-plane by requiring the Euclidean hodograph to satisfy

$$[x'(t)]^2 + [y'(t)]^2 = \rho(t)^2,$$

where ρ is a rational function. Under the Poincaré metric, the hyperbolic speed becomes

$$v_H(t) = \frac{\rho(t)}{y(t)},$$

and is therefore rational. We derive necessary and sufficient conditions for rational Pythagorean hodograph curves in Poincaré upper half-plane to possess the hyperbolic Pythagorean hodograph property and establish a characterization in terms of rational hodographs. Existence of hyperbolic Pythagorean hodograph curves of unit hyperbolic-speed is proved and their geometric structure is examined. Furthermore, the polynomial case is analyzed, revealing that nontrivial hyperbolic Pythagorean hodograph curves necessarily belong to the rational class. Explicit examples are presented to illustrate the theory. In brief, this work provides a hyperbolic extension of Farouki's classical theory of Pythagorean hodograph curves.

Keywords: Hyperbolic plane, Pythagorean hodograph curve, curve characterization.
2010 Mathematics Subject Classification: 14H50, 53A35.

References

[1] J. W. Anderson, Hyperbolic geometry, Springer, USA, 2005.

[2] R. T. Farouki, T. Sakkalis, Pythagorean hodographs, *IBM J. Res. Dev.* **34** (1990), 736–752.

[3] R. T. Farouki, The conformal map $z \to z^2$ of the hodograph plane, *Comput. Aided Geom. Design* **11** (1994), 363–390.

[4] R. T. Farouki, H. Pottmann, Polynomial and rational Pythagorean-hodograph curves reconciled, In: Proc. 6th IMA Conf. Math. Surfaces, London, September 1, 1994, pp. 355–378.

[5] R. T. Farouki, Pythagorean-hodograph curves, Springer, Berlin, Heidelberg, 2008.

[6] T. Sakkalis, R. T. Farouki, L. Vaserstein, Non-existence of rational arc length parameterizations for curves in $\mathbb{R}^n$, *J. Comput. Appl. Math.* **228** (2009), 494–497.

Curvatures of Dual Curves in 2-Dimensional Setting

Nursemin Çavdar, Muhittin Evren Aydın, Mahmut Ergüt

Department of Mathematics, Tekirdağ Namık Kemal Üniversitesi, Tekirdağ, Türkiye, nursemin_cavdar@hotmail.com
Department of Mathematics, Fırat University, Elazığ, Türkiye, meaydin@firat.edu.tr
Department of Mathematics, Tekirdağ Namık Kemal Üniversitesi, Tekirdağ, Türkiye, mergut@nku.edu.tr

Abstract

In this talk, we investigate curves in the dual plane parametrized by a real variable, from the viewpoint of equiaffine geometry. We also consider such curves in the dual Lorentz–Minkowski plane. When the curvature is constant, the corresponding results are presented. Non-trivial examples are also provided.

Keywords: dual plane, Lorentz-Minkowski plane, equiaffine curvature.
2020 Mathematics Subject Classification: 53A04, 53A15.

References

[1] B. Abalı A. Yücesan, Associated curves of a Frenet curve in the dual Lorentzian space, *Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat.* **71(1)** (2022), 285-304.

[2] N. Ayyıldız, A. C. Çöken, A. Yücesan, A characterization of dual Lorentzian spherical curves in the dual Lorentzian space, *Taiwan. J. Math.* **11(4)** (2007), 999-1018.

[3] M. A. Güngör, S. Ersoy, M. Tosun, Dual Lorentzian spherical motions and dual Euler–Savary formula, *Eur. J. Mech. A Solids* **28(4)** (2009), 820-826.

[4] R. López, Differential geometry of curves in dual space, *Vietnam J. Math.* (2025), https://doi.org/10.1007/s10013-025-00766-7.

[5] K. Nomizu, T.Sasaki, Affine Differential Geometry: Geometry of Affine Immersions, Cambridge Tracts in Mathematics 111, Cambridge University Press, Cambridge, (1994).

[6] G.R.Veldkamp, On the use of dual numbers, vectors and matrices in instantaneous, spatial kinematics, *Mech. Mach. Theory,* **11** (1976), 141-156.

Geometric Potential of Surfaces in Euclidean Spaces

Betül Bulca Sokur

Department of Mathematics, Bursa Uludağ University, Bursa, Türkiye,
bbulca@uludag.edu.tr

Abstract

In the present study, we considered skew curvatures of the surfaces to generate their geometric potentials. The method depends essentially on the mean and Gaussian curvatures and their principal curvatures. In quantum mechanics in the study of the dynamics of massive particle with mass m constrained to move on a surface. In such a case, the difference function of the squared mean curvature with the Gaussian curvature induces a geometric (scalar) potential. This potential appears in the Shröndiger type equations. Considering the skew curvatures of the rotational surfaces, some results on the meridian curves are obtained. The term "geometric potential" in the context of rotational surfaces often relates to the functional or energy that describes certain properties of these surfaces. The geometric potential of rotational surfaces encompasses various functionals and energies related to their geometry. Furthermore, the geometric potentials of level surfaces and generalized helicoidal surfaces are calculated. Finally we discuss the some applications of these types of surfaces in quantum mechanics.

References

[1] V. Atanasov, R. Dandoloff, Curvature induced quantum potential on deformed surfaces, *Phys. Lett. A* **371** (2007), 118–123.

[2] V. Atanasov, R. Dandoloff, Curvature-induced quantum behaviour on a helical nanotube, *Phys. Lett. A* **372** (2008), 6141–6144.

[3] B. Y. Chen, On the difference curvature of surfaces in Euclidean space, *Math. J. Okayama Univ.* **14** (1970), 153–157.

[4] R. Dandoloff, A. Saxena, B. Jensen, Geometry-induced potential on a two-dimensional section of a wormhole: Catenoid, *Phys. Rev. A* **81** (2010), 014102.

[5] R. Lopez, A. Pampano, Classification of rotational surfaces in Euclidean space satisfying a linear relation between their principal curvatures, *Math. Nachr.* **293** (2020), 735–753.

[6] R. Lopez, A. Pampano, Classification of rotational surfaces with constant skew curvature in 3-space forms, *J. Math. Anal. Appl.* **489** (2020), 1–19.

[7] R. Lopez, A. Pampano, Rotational surfaces of constant astigmatism in space forms, *J. Math. Anal. Appl.* **483** (2020), 1–21.

[8] M. Toda, A. Pigazzini, A note on the class of surfaces with constant skew curvature, *J. Geom. Symmetry Phys.* **46** (2017), 51–58.

[9] L. C. B. da Silva, Surfaces of revolution with prescribed mean and skew curvatures in Lorentz-Minkowski space, *Tohoku Math. J.* **73** (2021), 317–339.

Generalized Connections and $*$-Ricci–Bourguignon Solitons on LP-Sasakian Manifolds

Murat Altunbaş, Ayşe Karanlık Akpınar

Department of Mathematics, Faculty of Arts and Sciences, Erzincan Binali Yıldırım University, Erzincan, Turkey, maltunbas@erzincan.edu.tr

Department of Mathematics, Graduate School of Natural and Applied Sciences, Erzincan Binali Yıldırım University, Erzincan, Turkey, aysekrnlk@hotmail.com

Abstract

In this study, we define and explore a general connection on Lorentzian Para-Sasakian (LP-Sasakian) manifolds that generalizes several well-known connections, including the quarter-symmetric metric, Schouten–Van Kampen, Tanaka–Webster and Zamkovoy connections. Utilizing this generalized framework, we investigate the properties of $*$-Ricci–Bourguignon solitons and examine the behavior of the T-curvature tensor. Finally, we provide a concrete 3-dimensional LP-Sasakian manifold example to illustrate and validate our theoretical results.

Keywords: $*$-Ricci–Bourguignon soliton, LP-Sasakian manifold, general connection, T-curvature tensor, symmetrized covariant derivative.
2010 Mathematics Subject Classification: 53C15, 53C25, 53C44.

References

[1] A. Biswas, K. K. Baishya, A general connection on Sasakian manifolds and the case of almost pseudo symmetric Sasakian manifolds, *Sci. Stud. Res. Ser. Math. Inform.* **29** (2019), 59–72.

[2] A. Biswas, K. K. Baishya, Study on generalized pseudo (Ricci) symmetric Sasakian manifold admitting general connection, *Bull. Transilv. Univ. Braşov* **12** (2019), 233–246.

[3] R. Kumar, L. Chawngthu, O. Bahadır, M. A. Khan, Geometry of LP-Sasakian manifolds admitting a general connection, *Mathematics* **13** (2025), 902.

Twin-Conformal Collinations

Sibel Turanlı, Aydın Gezer

Department of Mathematics, Erzurum Technical University, Erzurum, Türkiye,
sibel.turanli@erzurum.edu.tr
Department of Mathematics, Atatürk University, Erzurum, Türkiye,
aydingzr@gmail.com

Abstract

Affine conformal Killing vector fields were introduced by Katzin, Levine, and Davis, and are characterized by the condition $L_\xi \nabla = d\rho \otimes I + I \otimes d\rho - g \otimes \nabla\rho$, which is equivalent to $L_\xi g = 2\rho g + K$, $\nabla K = 0$, where ρ is a smooth function and K is a symmetric parallel $(0,2)$-tensor field. This structure leads naturally to the notion of conformal collineations, which play an important role in differential geometry and general relativity. Motivated by this classical framework, we introduce a new symmetry notion on three-dimensional Lorentzian Walker manifolds admitting a parallel degenerate line field, called twin conformal collineations. In this setting, the role of the symmetric tensor field K is replaced by the twin metric associated with the Walker structure. We consider vector fields satisfying the generalized conformal-type equation $L_\xi g = 2\rho g + \tilde{g}$, where ρ is an arbitrary smooth function and $\tilde{g}$ denotes the twin metric of the Lorentzian Walker manifold. By computing the Lie derivative of the Walker metric, we obtain a system of partial differential equations characterizing such vector fields. This system is analyzed under several geometric conditions. Explicit solutions are obtained under special assumptions, and further results are derived for strictly Walker three-manifolds. In addition, the conformally flat strictly Walker case is investigated, revealing further geometric properties of twin conformal collineations.

Keywords: Killing vector fields, Lorentzian Walker manifolds.
2010 Mathematics Subject Classification: 53C50, 53C21, 53C15.

References

[1] M. M. Akbar, E. Woolgar, Ricci solitons and Einstein-scalar field theory, *Class. Quantum Grav.* **26** (2009), 055015.

[2] M. Brozos-V'azquez, G. Calvaruso, E. Garc'ıa-R'ıo, S. Gavino-Fern'andez, Three-dimensional Lorentzian homogeneous Ricci solitons, *Israel J. Math.*, to appear.

[3] M. Chaichi, E. Garc'ıa-R'ıo, M. E. V'azquez-Abal, Three-dimensional Lorentz manifolds admitting a parallel null vector field, *J. Phys. A: Math. Gen.* **38** (2005), 841–850.

[4] A. G. Walker, Canonical form for a Riemannian space with a parallel field of null planes, *Quart. J. Math. Oxford Ser.* **1** (1950), 69–79.

Some Gradient Solitons on Siklos Spacetimes

Sinem Güler

Department of Industrial Engineering, Istanbul Sabahattin Zaim University, Istanbul, Türkiye,
sinem.guler@izu.edu.tr

Abstract

Siklos metrics are exact solutions of the Einstein's field equations. In the Petrov classifications, they correspond to type N spacetimes with cosmological constant $\Lambda < 0$. Moreover, they admit a null non-twisting Killing fields. More formally, Siklos metrics are typically given by

$$g = -\frac{3}{\Lambda x_3^2}(2dx_1dx_2 + H(x_2, x_3, x_4)dx_2 + dx_3^2 + dx_4^2)$$

with respect to a local coordinate system (x_1, x_2, x_3, x_4), for some defining smooth function $H = H(x_2, x_3, x_4)$. For instance, Siklos spacetimes are known to be Einstein spaces conformally related to flat pp-wave geometries, [1]. For further details, we refer to [2, 3].

The aim of this study is to investigate the existence of different types of gradient solitons by solving the systems of differential equations arising from their fundamental equations.

Keywords: Siklos metric, gradient Ricci soliton, gradient Yamabe soliton.
2010 Mathematics Subject Classification: 53C50, 53B30.
Acknowledgements: This work is supported by Istanbul Sabahattin Zaim University under the BAP project (Project Number: 2025-BAP-400-062).

References

[1] S. T. C. Siklos, Galaxies, axisymmetric systems and relativity, In: MacCallum, M.A.H. (ed.) Lobatchevski plane gravitational waves, p. 247. Cambridge University Press, Cambridge, 1985.

[2] J. Podolský, Interpretation of the Siklos solutions as exact gravitational waves in the anti-de Sitter universe, *Class. Quantum Grav.* **15** (1998), 719–733.

[3] G. Calvaruso, M. Kaflou and A. Zaeim, On the symmetries of Siklos spacetimes, *Gen. Relativ. Gravit.* **54** (2022) 60.

Pedal Curves according to the Modified Orthogonal Frame in Euclidean 3-Space

Burçin Saltık Baek, Nural Yüksel, Esra Damar

Department of Mathematics, Erciyes University, Kayseri, Türkiye, burcinsaltik@erciyes.edu.tr

Department of Mathematics, Erciyes University, Kayseri, Türkiye, yukseln@erciyes.edu.tr

Department of Motor Vehicles and Transportation Technologies, Vocational School of Technical Sciences, Hitit University, Çorum, Türkiye, esradamar@hitit.edu.tr

Abstract

In this paper, we introduce and investigate three new pedal-type curves generated by the modified orthogonal frame of a regular space curve in Euclidean three-space. More precisely, the modified T-pedal, modified N-pedal and modified B-pedal curves are defined as the loci of the orthogonal projections of a fixed point onto the lines spanned by the modified frame vectors. For each curve, the Frenet frame elements, curvature and torsion are obtained in terms of the modified orthogonal frame apparatus of the original curve. Several geometric characterizations are then derived, including regularity, planarity, general helix and circular helix conditions. Furthermore, the relations among the three modified pedal curves are examined. It is shown that the three pedal curves satisfy a simple vector identity involving the original curve and the fixed pedal point. Finally, explicit examples based on a circular helix are visualized.

Keywords: Pedal curve, modified orthogonal frame.
2010 Mathematics Subject Classification: 53A04

References

[1] B. Bükcü, M. K. Karacan, On the modified orthogonal frame with curvature and torsion in 3-space, *Math. Sci. Appl. E-Notes* **4** (2016), 184–188.

[2] Y. Li, D. Pei, Pedal curves of frontals in the Euclidean plane, *Math. Methods Appl. Sci.* **41** (2018), 1988–1997.

[3] D. Canlı, S. Şenyurt, F. Ertem Kaya, L. Grilli, The pedal curves generated by alternative frame vectors and their Smarandache curves, *Symmetry* **16** (2024), 1012.

[4] A. Elsharkawy, H. K. Elsayied, M. E. Desouky, C. Cesarano, T-pedal ruled surface with the Frenet frame of the original curve in E^3, *AIMS Math.* **10** (2025), 25606–25623.

[5] A. Elsharkawy, H. Elsayied, M. Desouky, C. Cesarano, A. Gasymov, A geometric study of ruled surfaces generated by Frenet vectors via N-pedal curves in E^3, *Azerbaijan J. Math.* **16** (2026), 3–15.

Grey Network Games and Equal Surplus Sharing in Regional Water Transmission Under Uncertainty

Halil İbrahim Dönmez, Mehmet Onur Olgun, Sırma Zeynep Alparslan Gök

Graduate School of Natural and Applied Sciences, Süleyman Demirel University, Isparta, Türkiye, hidonmez35@gmail.com

Department of Industrial Engineering, Faculty of Engineering and Natural Sciences, Süleyman Demirel University, Isparta, Türkiye, onurolgun@sdu.edu.tr

Department of Mathematics, Faculty of Engineering and Natural Sciences, Süleyman Demirel University, Isparta, Türkiye, sirmagok@sdu.edu.tr

Abstract

In this study, multi-owner maximum flow problems defined under grey uncertainty are investigated within the framework of cooperative game theory [1]. In this context, egalitarian distribution approaches are comprehensively addressed by considering the flow and risk allocation dimensions in flow problems. Grey interval numbers [3] are used to model situations where arc capacities cannot be known precisely in real-life logistics and infrastructure networks. In this context, we aim to overcome the limitations of both deterministic and probabilistic approaches.

Based on a single-source, single-sink and multi-owner network structure [2], where arc capacities are expressed by grey interval numbers, two separate games are formulated for all possible coalitions: the grey maximum flow game $\langle N, v' \rangle$ and the grey risk game $\langle N, c' \rangle$. The grey characteristic function of each coalition is obtained using the BWC algorithm [5] and the Grey REILP method [6]. Through this method, a quantitative balance is established between risk and system return.

Four Equal Surplus Sharing Approaches, namely the Grey Banzhaf value, Grey CIS-value ($\mathcal{G}CIS$), Grey ENSC-value ($\mathcal{G}ENSC$), and Grey ED-value ($\mathcal{G}ED$) [4], are applied to both games. The results are evaluated comparatively in terms of efficiency, individual rationality, and coalitional rationality criteria. The findings demonstrate that the $\mathcal{G}ENSC$-value lies within the grey core and ensures coalition stability. The results make significant contributions to cooperative models for designing fair, stable, and risk-sensitive collaboration in infrastructure and logistics networks under grey uncertainty.

Keywords: Grey numbers, Cooperative game theory, Maximum flow problem, Risk allocation, Grey uncertainty.
2020 Mathematics Subject Classification: Primary: 91A12; Secondary: 91A43.

References

[1] S. Z. Alparslan Gök, O. Branzei, R. Branzei, Cooperative interval games arising from airport situations with interval data, CentER Discussion Paper, Tilburg University, 2008.

[2] A. Baykasoğlu, B. Kubur Özbel, Explicit flow-risk allocation for cooperative maximum flow problems under interval uncertainty, *Oper. Res.* **22** (2019), 1165–1198.

[3] J. L. Deng, Control problems of grey systems, *Syst. Control Lett.* **1** (1982), 288–294.

[4] H. İ. Dönmez, M. O. Olgun, S. Z. Alparslan Gök, Equal surplus sharing in grey inventory games, *Alphanumeric J.* **12** (2024), 215–226.

[5] T. Shaocheng, Interval number and fuzzy number linear programmings, *Fuzzy Sets Syst.* **66** (1994), 301–306.

[6] R. Zou, Y. Liu, L. Liu, H. Guo, REILP approach for uncertainty-based decision making in civil engineering, *J. Comput. Civ. Eng.* **24** (2010), 357–364.

Some Special Collineations on an Almost Product Lorentzian Walker Three-Manifold

Büşra Aktaş, Aydın Gezer
Department of Mathematics, Kırıkkale University, Kırıkkale, Turkiye,
baktas6638@gmail.com
Department of Mathematics, Atatürk University, Erzurum, Turkiye,
aydingzr@gmail.com

Abstract

In this study, we equip a three-dimensional Lorentzian Walker manifold (M^3, g_f) with an almost product structure J, thereby obtaining the almost product Lorentzian Walker three-manifold (M^3, J, g_f). This setting gives rise to a twin metric G_f, whose geometric properties constitute the main focus of the paper. On the almost product Lorentzian strictly Walker three-manifold (M^3, J, g_f), we establish that the metric G_f admits recurrent curvature and derive complete characterizations of several important symmetry classes associated with G_f. Moreover, a collection of explicit examples arising from special classes of locally homogeneous Lorentzian manifolds with recurrent curvature is presented.

Keywords: Walker Manifolds, Product Structure, Collineations.
2020 Mathematics Subject Classification: 53C50, 53C21.

References

[1] A. G. Walker, Canonical form for a Riemannian space with a parallel field of null planes, *Quart. J. Math. Oxford Ser.* **1** (1950), 69–79.

[2] E. Garcia-Rio, P. B. Gilkey, S. Nikcevic, Homogeneity of Lorentzian three-manifolds with recurrent curvature, *Math. Nachr.* **287** (2014), 32–47.

[3] G. Calvaruso, A. Zaeim, Symmetries of Lorentzian three-manifolds with recurrent curvature, *SIGMA Symmetry Integrability Geom. Methods Appl.* **12** (2016), Paper No. 063.

[4] M. Brozos-Vazquez, E. Garcia-Rio, P. Gilkey, S. Nikcevic, R. Vazquez-Lorenzo, The geometry of Walker manifolds, Synthesis Lectures on Mathematics and Statistics, Vol. 5, Morgan & Claypool Publishers, Williston, VT, 2009.

[5] M. Chaichi, E. Garcia-Rio, M. E. Vazquez-Abal, Three-dimensional Lorentz manifolds admitting a parallel null vector field, *J. Phys. A: Math. Gen.* **38** (2005), 841–850.

[6] M. Tsamparlis, P. S. Apostolopoulos, Ricci and matter collineations of locally rotationally symmetric spacetimes, *Gen. Relativity Gravitation* **36** (2004), 47–69.

On the Marginally Trapped Surfaces of a Warped Space-Time

Nurettin Cenk Turgay
Department of Mathematics, Istanbul Technical University, Istanbul, Türkiye,
turgayn@itu.edu.tr

Abstract

Let $L_1^3(c) = (\Omega_c, g_c)$ denote the three-dimensional Lorentzian space form of constant sectional curvature c, where

$$g_c = \frac{1}{\left(1 + \frac{c}{4}(-t^2 + x^2 + y^2)\right)^2} \left(-dt^2 + dx^2 + dy^2\right),$$

defined on

$$\Omega_c = \left\{(t, x, y) \in \mathbb{R}^3 : 1 + \frac{c}{4}(-t^2 + x^2 + y^2) > 0\right\}.$$

Let $I \subset \mathbb{R}$ be an open interval and let $f : I \to \mathbb{R}$ be a positive smooth function. We consider the warped space-time

$$\widetilde{M} = L_1^3(c)_f \times I = (\Omega_c \times I, \widetilde{g}),$$

endowed with the metric

$$\widetilde{g} = f(z)^2 g_c + dz^2.$$

Note that $\widetilde{M}$ admits a natural foliation by space-like hypersurfaces $\Omega_c \times \{z\}$, each of which is homothetic to $L_1^3(c)$.

A space-like surface immersed in $\widetilde{M}$ is called *marginally trapped* if its mean curvature vector field is lightlike at every point.

In this talk, we establish several new results on the geometry of marginally trapped surfaces in $\widetilde{M}$.

Keywords: Lorentzian space forms, warped product space-time, marginally trapped surfaces.
2010 Mathematics Subject Classification: 53C50, 53B30.
Acknowledgment: The author acknowledges ChatGPT for assistance in improving the language and presentation of the abstract.

References

[1] B. O'Neill, Semi-Riemannian Geometry with Applications to Relativity, *Academic Press*, (1983).

[2] R. L. Bishop and B. O'Neill, Manifolds of negative curvature, *Trans. Amer. Math. Soc.* **145** (1969), 1–49.

[3] I. Booth, K. T. B. Chan, R. A. Hennigar, H. Kunduri and S. Muth Exotic marginally outer trapped surfaces in rotating spacetimes of any dimension, *Class. Quantum Grav.* **40** (2023), 095010.

Recent Developments of Bertrand Curves in Minkowski Space-time

Kazım İlarslan

Department of Mathematics, Kırıkkale University, Kırıkkale, TÜRKİYE
kilarslan@yahoo.com

Abstract

Bertrand curves are one of the classical topics in differential geometry and have been extensively studied in both Euclidean and Lorentzian geometries. In Minkowski space-time, the causal character of curves leads to the emergence of different Bertrand correspondences involving non-null, pseudo-null, and Cartan null curves.

In this talk, we present recent developments in the theory of Bertrand curves in Minkowski space-time. After a brief review of the classical theory, new results concerning the existence and characterization of Bertrand pairs for curves with different causal characters are discussed.

Keywords: Bertrand curves, Cartan null curves, pseudo-null curves, Minkowski space-time.
2010 Mathematics Subject Classification: 53C50, 53C40.

References

[1] O. Keçilioğlu, K. İlarslan, M. K. Uçar, Bertrand curves in Minkowski space-time: A new approach, *Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat.*, Accepted, 2026.

[2] K. İlarslan, O. Keçilioğlu, M. B. Özcan, Y. Li, New characterization results on Bertrand mates of Cartan null curves in Minkowski space-time, Submitted, 2026.

[3] H. Matsuda, S. Yorozo, Notes on Bertrand curves, *Yokohama Math. J.* **50** (2003), 41–58.

[4] A. Uçum, O. Keçilioğlu, K. İlarslan, Generalized Bertrand curves with spacelike $(1, 3)$-normal plane in Minkowski space-time, *Turkish J. Math.* **40** (2016).

Some Properties of Weak Almost Contact Manifolds

Erol Yaşar, Şemsi Eken Meriç, Erol Kılıç

Department of Mathematics, Faculty of Science, Mersin University, Mersin, Türkiye
yerol@mersin.edu.tr
Department of Mathematics, Faculty of Science, Mersin University, Mersin, Türkiye
semsieken@mersin.edu.tr
Department of Mathematics, Faculty of Science, İnönü University, Malatya, Türkiye
erol.kilic@inonu.edu.tr

Abstract

In this paper, we focus on weak almost contact structures and study some geometric properties of them like their splitting, paralellism and so on. Here, we find some interesting results and present some examples for weak almost contact manifolds.

Keywords: Almost contact structure, Killing vector field
2010 Mathematics Subject Classification: 53C15, 53C25.

References

[1] D. Blair, Riemannian Geometry of Contact and Symplectic Manifolds, Springer, 2010.

[2] A. Bejancu, CR-Submanifolds, D. Reidel Pub. Co.; Sold and distributed in the U.S.A. and Canada by Kluwer Academic Publishers, 1986.

[3] D. S. Patra, V. Rovenski, On the rigidity of the Sasakian structure and characterization of cosymplectic manifolds, *Diff. Geom. Appl.* **90** (2023) 102043.

[4] V. Rovenski, R. Wolak, New metric structures on $\mathfrak{g}-$foliations, *Indag. Math.* **33**(3) (2022) 518 - 532.

[5] V. Rovenski, Weak nearly Sasakian and weak nearly cosymplectic manifolds. *Mathematics* **11**(20), (2023) 4377.

[6] V. Rovenski, Geometry of a weak para-f-structure. *U.P.B. Scientific Bulletin, Series A: Appl. Math. and Phys.* **85**(4), (2023) 11-20.

[7] V. Rovenski, Foliated structure of weak nearly Sasakian manifolds, *Annali di Matematica Pura ed Applicata,* **203**(6), (2024) 2641-2652.

Sequential Framed Direction Curves of Framed Curves and Their Characterizations

Medine Bakraç, Mustafa Düldül

Department of Mathematics, Yildiz Technical University, İstanbul, Türkiye
bakracmedine2@gmail.com
Department of Mathematics, Yildiz Technical University, İstanbul, Türkiye
mduldul@yildiz.edu.tr

Abstract

In this study, sequential framed direction curves of framed generalized principal-direction curves, framed generalized binormal-direction curves and framed generalized Darboux-direction curves of framed curves are studied in the Euclidean 3-space. Some characterizations are given for the obtained curves when the given curve is a framed slant helix, a framed clad helix, or a framed g-clad helix.

Keywords: Framed curve, framed slant helix, framed clad helix, framed g-clad helix, framed direction curve.
2020 Mathematics Subject Classification: 53A04.

References

[1] Ç. Camci, B. Y. Chen, K. İlarslan, A. Uçum, Sequential natural mates of Frenet curves in Euclidean 3-space, *J. Geom.* **112** (2021), 46.

[2] M. Dündül, Z. Bülbül, Integral curves connected with a framed curve in 3-space, *Honam Math. J.* **45** (2023), 130–145.

[3] M. Mak, Framed clad helices in Euclidean 3-space, *Filomat* **37** (2023), 9627–9640.

[4] O. Z. Okuyucu, M. Canbirdi, Framed slant helices in Euclidean 3-space, *Adv. Difference Equ.* **2021** (2021), Article No. 504.

[5] S. Honda, M. Takahashi, Framed curves in the Euclidean space, *Adv. Geom.* **16** (2016), 265–276.

[6] T. Takahashi, N. Takeuchi, Clad helices and developable surfaces, *Bull. Tokyo Gakugei Univ. Div. Nat. Sci.* **66** (2014), 1–9.

[7] Y. Li, M. Mak, Framed natural mates of framed curves in Euclidean 3-space, *Mathematics* **11** (2023), 3571.

[8] Y. Wang, D. Pei, R. Gao, Generic properties of framed rectifying curves, *Mathematics* **7** (2019), 37.

A Multiplicative Calculus Perspective on AW(k)-Type Curves

Tugay Türkmen, Serpil Ünal, Semra Kaya

Department of Mathematics, Faculty of Engineering and Natural Sciences, Usak University, 64000, Usak, Türkiye, ttugayturkmen@outlook.com
Department of Mathematics, Faculty of Engineering and Natural Sciences, Usak University, 64000, Usak, Türkiye, serpil.unal@usak.edu.tr
Department of Mathematics, Faculty of Engineering and Natural Sciences, Usak University, 64000, Usak, Türkiye, semra.kaya@usak.edu.tr

Abstract

In this paper, it is aimed to obtain characterizations by considering Multiplicative AW(k)-type curves in geometric space (Multiplicative Analysis). Also, geometric properties and original results related to the concept of Multiplicative AW(k)-type curves are obtained.

Keywords: Multiplicative calculus, AW(k) type curve, helix.
2010 Mathematics Subject Classification: 11A05, 53A04, 53A35.

References

[1] V. Volterra, B. Hostinsky, Operations infinitesimales lineares, Herman, Paris, 1938.

[2] M. Grossman, R. Katz, Non-Newtonian calculus, 1st ed., Lee Press, Pigeon Cove, Massachusetts, 1972.

[3] M. Grossman, Bigeometric calculus: A system with a scale-free derivative, Archimedes Foundation, Massachusetts, 1983.

[4] N. Yalçın, F. Celik, Solution of multiplicative homogeneous linear differential equations with constant exponentials, *New Trends Math. Sci.* **6** (2018), 58–67.

[5] M. Rybaczuk, W. Zielinski, The concept of physical and fractal dimension I. The projective dimensions, *Chaos Solitons Fractals* **12** (2001), 2517–2535.

[6] D. Aniszewska, Multiplicative Runge-Kutta methods, *Nonlinear Dyn.* **50** (2007), 262–272.

[7] S. G. Georgiev, K. Zennir, Multiplicative differential calculus, 1st ed., Chapman and Hall/CRC, New York, 2022.

[8] S. G. Georgiev, Multiplicative differential geometry, 1st ed., Chapman and Hall/CRC, New York, 2022.

[9] S. G. Georgiev, K. Zennir, A. Boukarou, Multiplicative analytic geometry, 1st ed., Chapman and Hall/CRC, New York, 2022.

[10] S. K. Nurkan, I. Gurgil, M. K. Karacan, Vector properties of geometric calculus, *Math. Methods Appl. Sci.* (2023), 1–20.

[11] M. E. Aydin, A. Has, B. Yilmaz, A non-Newtonian approach in differential geometry of curves: multiplicative rectifying curves, *Bull. Korean Math. Soc.* **61** (2024), 849–866.

A Note on Weak Almost Contact Manifolds

Şemsi Eken Meriç, Sadık Keleş, Rıfat Güneş
Department of Mathematics, Faculty of Science, Mersin University, Mersin, Türkiye
semsieken@mersin.edu.tr
Department of Mathematics, Faculty of Science, İnönü University, Malatya, Türkiye
sadik.keles@inonu.edu.tr
Department of Mathematics, Faculty of Science, İnönü University, Malatya, Türkiye
rifat.gunes@inonu.edu.tr

Abstract

In this paper, we study on a weak almost contact manifolds and give some characterizations for them. Here, we investigate torse-forming vector field on such manifolds and obtain some results.

Keywords:Almost contact structure, torse-forming vector field.
2010 Mathematics Subject Classification: 53C15, 53C25

References

[1] D. Blair, Riemannian Geometry of Contact and Symplectic Manifolds, Springer, 2010.

[2] D. S. Patra, V. Rovenski, On the rigidity of the Sasakian structure and characterization of cosymplectic manifolds, *Diff. Geom. Appl.* **90** (2023) 102043.

[3] V. Rovenski, R. Wolak, New metric structures on $\mathfrak{g}-$foliations, *Indag. Math.* **33**(3) (2022) 518 - 532.

[4] V. Rovenski, Weak nearly Sasakian and weak nearly cosymplectic manifolds. *Mathematics* **11**(20), (2023) 4377.

[5] V. Rovenski, Geometry of a weak para-f-structure. *U.P.B. Scientific Bulletin, Series A: Appl. Math. and Phys.* **85**(4), (2023) 11-20.

[6] V. Rovenski, Foliated structure of weak nearly Sasakian manifolds, *Annali di Matematica Pura ed Applicata,* **203**(6), (2024) 2641-2652.

A Smoothed Objective Penalty Function Method for Solving Constrained Optimization Problems

Nurullah Yılmaz, Dilara Kocatürk

Department of Mathematics, Suleyman Demirel University,Isparta, Türkiye, nurullahyilmaz@sdu.edu.tr

Graduate School of Natural and Applied Sciences, Suleyman Demirel University, Isparta, Türkiye, dilaraakocaturkk@gmail.com

Abstract

Penalty function techniques are commonly used in the literature to convert nonlinear constrained optimization problems into unconstrained formulations by incorporating the constraints into the objective penalty function. In this study, the objective penalty technique and exact penalty property are investigated. Applying derivative-based algorithms has been difficult due to non-differentiability of the original function. For the objective penalty function, a new smoothing function is proposed. This approach has made the penalty function smooth and continuously differentiable, resulting in a second-order function of the C^2 class. This transformation allows derivative-based problem-solving.

In the theoretical framework of this research, the fundamental relationships between the smoothed problem and the original problem have been thoroughly analyzed. As a result of comprehensive mathematical investigations, it has been theoretically proven that a global optimal solution derived from the smoothed penalty problem constitutes an approximate global optimal solution for the original non-smooth penalty problem. Consequently, a new optimization algorithm combined with a newly smoothed objective penalty function has been developed. The convergence analysis of the algorithm has been presented. Finally, the efficiency of the proposed algorithm has been substantiated through both theoretical and numerical evaluations, which have exhibited the robust computational performance and reliability of the method.

Keywords: Constrained optimization, exact penalty function, smoothed penalty function.
2010 Mathematics Subject Classification: 90C30, 65D10, 65K05.

References

[1] Z. Meng, Q. Hu, C. Dang & X. Yang, An objective penalty function method for nonlinear programming, *Appl. Math. Lett.* **17**(6) (2004), 683-689.

[2] X. Xu, Z. Meng, J. Sun, L. Huang & R. Shen, A second-order smooth penalty function algorithm for constrained optimization problems, *Comput. Optim. Appl.* **55**(1) (2013), 155-172.

[3] J. Qiu, J. Yu & S. Lian, Smoothing approximation to the new exact penalty function with two parameters, *Asia-Pac. J. Oper. Res.* **38**(05) (2021), 2140010.

A Study on Tangent Ruled Surfaces of Associated Curves in the Modified Orthogonal Frame

Nural Yüksel, Nurdan Oğraş, Süleyman Şenyurt

Department of Mathematics, Erciyes University, Kayseri, Turkey yukseln@erciyes.edu.tr
Graduate School of Natural and Applied Sciences, Erciyes University, Kayseri, Turkey, nurdanogras@gmail.com
Department of Mathematics, Ordu University, Ordu, Turkey, senyurtsuleyman52@gmail.com

Abstract

In this study, tangent ruled surfaces generated from associated curves are investigated within the Modified Orthogonal Frame.The associated curves are considered as base curves, while the tangent vector field acts as the director vector of the surface. The geometric properties of these surfaces are examined by computing the Gaussian curvature, mean curvature, and shape operator. In addition, necessary conditions for developability and minimality are obtained. The singular points and parameter curves are also analyzed with respect to their asymptotic and geodesic properties. Several examples and graphical illustrations are presented to support the theoretical results.

Keywords: Associated curves, ruled surfaces, modified orthogonal frame, Fresnel curve.
2010 Mathematics Subject Classification: 53A05.

References

[1] B. Bükcü, M. K. Karacan, On the modified orthogonal frame with curvature and torsion in 3-space, *Math. Sci. Appl. E-Notes* **4** (2016), 184–188.

[2] M. P. do Carmo, Differential geometry of curves and surfaces, Prentice-Hall, New Jersey, 1976.

[3] M. Özdemir, Diferansiyel geometri, Altın Nokta Yayınevi, İzmir, 2021.

[4] B. S. Baek, E. Damar, N. Oğraş, N. Yüksel, Ruled surfaces of adjoint curve with the modified orthogonal frame, *J. New Theory* **49** (2025), 69–82.

[5] S. Şenyurt, D. Canlı, K. H. Ayvacı, Associated curves from a different point of view in E^3, *Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat.* **71** (2022), 826–845.

[6] S. Şenyurt, S. G. Mazlum, D. Canlı, E. Can, Some special Smarandache ruled surfaces according to alternative frame in E^3, *Maejo Int. J. Sci. Technol.* **17** (2023), 1–15.

A New One-Parameter Filled Function Algorithm with Initialization Strategies for Global Optimization

Nurullah Yılmaz, Eylül Ayyıldız

Department of Mathematics, Suleyman Demirel University,Isparta, Türkiye, nurullahyilmaz@sdu.edu.tr

Graduate School of Natural and Applied Sciences, Suleyman Demirel University, Isparta, Türkiye, ayyildizeylul8@gmail.com

Abstract

Global optimization problems frequently have multiple local extrema, which can trap solution techniques at these minima and prevent convergence to the global minimum. To address this issue, filled function techniques have been proposed in the literature. These techniques have been designed to facilitate the transition of algorithms between different basins in the solution space and to reduce their effects by penalizing local minima. In this paper, a new one-parameter continuously differentiable filled function is proposed.

As is known, the practical success of approaches such as the filled function method largely depends on the starting points that the algorithm uses in the process of exploring the search space, moving from the existing local minimum to a lower local minimum. The distribution and choice of initialization points employed in the algorithmic stage have critical importance. To obtain better results, a new global optimization algorithm is developed based on different initialization techniques, such as grid sampling and random sampling. The algorithms have been coded in MATLAB and tested on standard global optimization problems. Finally, numerical results have been reported, and the computational efficiency and success of the new filled function algorithms have been analyzed.

Keywords: Global optimization, filled function method, random sampling, grid sampling.

2010 Mathematics Subject Classification: 90C26, 90C30.

References

[1] G. Renpu, The theory of filled function method for finding global minimizers of nonlinearly constrained minimization problems, *J. Comput. Math.*, (1987), 1-9.

[2] Y. Yang, Y. Shang, A new filled function method for unconstrained global optimization, *Appl. Math. Comput.* **173**(1) (2006), 501-512.

[3] P. Jaiswal, J. Larson, Multistart algorithm for identifying all optima of nonconvex stochastic functions, *Optim. Lett.* **18**(6) (2024), 1335-1360.

Prolongations of Almost Hsu-Golden Structures to Tangent Bundles

Tuğba Tamirci, Mustafa Özkan

Department of Mathematics, Faculty of Science, Gazi University, Ankara, Türkiye,
tugbatamirci@gmail.com
Department of Mathematics, Faculty of Science, Gazi University, Ankara, Türkiye,
ozkanm@gazi.edu.tr

Abstract

In this study, we studied the complete and horizontal lifts of the almost Hsu-golden structure, that is, a polynomial structure with the structure polynomial $Q(x) = x - \frac{1-\alpha^r}{4}$ where $r \in \mathbb{Z}$, $\alpha \in \mathbb{C} \setminus \{0\}$ in the tangent bundle. We investigate integrability and parallelism of almost Hsu-golden structures in the tangent bundle.

Keywords: Hsu structure, almost Hsu-golden structure, prolongations, tangent bundle, complete lift, horizontal lift, integrability.
2010 Mathematics Subject Classification: 53C05, 53C15, 57N16.

References

[1] B. Ay, M. Özkan, Almost Hsu-golden structure on manifolds, *Turk. J. Math. Comput. Sci.* **17(1)** (2025), 264–274.

[2] A. Bejancu, H.R. Farran, Foliations and Geometric Structures, *Mathematics and its Applications*, 580, Springer, 2006.

[3] L. Bisht, S. Shanker, On recurrent Hsu-structure manifold, *Int. J. Eng. Res. Appl. (Ijera)*, **2(6)**, 2012, 1323–1328.

[4] L. Bisht, On Hsu-structure manifold, Nijenhuis tensor, *Int. J. Adv. Res. Technol.* **2**, 2013, 87–96.

[5] L.S. Das, Prolongations of F-structure to the tangent bundle of order 2, *Internat. J. Math. Sci.* **16(1)** (1993), 201–204.

[6] U. De, A. Gezer, C. Karaman, Results concerning semi-symmetric metric F-connections on the Hsu-B manifolds, *Commun. Korean Math. Soc.* **38** (2023), 837–846.

[7] T. Omran, A. Sharffuddin, S. I. Husain, Lifts of structures on manifolds, *Publications De L'institut Math.* **36(50)** (1984), 93–97.

[8] M. Özkan, Prolongations of Golden structures to tangent bundles, *Diff. Geom. Dyn. Syst.* **16** (2014), 227–238.

[9] M. Özkan, F. Yılmaz, Prolongations of golden structures to tangent bundles of order r, *Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat.* **65(1)** (2016), 35–47.

[10] K. Yano, S. Ishihara, Tangent and Cotangent Bundles, Marcel Dekker Inc., New York, 1973.

On Classifications of Almost Poly-Norden Pseudo-Riemannian Manifolds

Neslihan Yıldırım, Mustafa Özkan

Department of Mathematics, Graduate School of Natural and Applied Sciences, Gazi University, Ankara, Türkiye, neslihanyildirim@gazi.edu.tr
Department of Mathematics, Faculty of Science, Gazi University, Ankara, Türkiye, ozkanm@gazi.edu.tr

Abstract

The classification of almost Norden manifolds has been studied using both the Levi-Civita connection and the canonical connection. The interpretation of the canonical connection as a well-adapted connection has led to analogous classifications for almost Norden golden manifolds. Motivated by these classifications and the introduction of poly-Norden manifolds, we investigate the classification of almost poly-Norden pseudo-Riemannian manifolds.

Keywords: Almost Norden manifold, almost poly-Norden manifold, classification.
2010 Mathematics Subject Classification: 53C15, 53C05, 53C07.

References

[1] B. Şahin, Almost poly-Norden manifolds, *Int. J. Maps Math.* **1** (1) (2018), 68–79.

[2] Ç. Karaman, Z. Topuz, On holomorphic poly-Norden manifolds, *Turk. J. Math.* **44** (4) (2020), 1388–1400.

[3] E. Kılıç, T. Acet, and S. Yüksel Perktaş, On lightlike submanifolds of poly-Norden semi-Riemannian manifolds, *Turk. J. Math. Comput. Sci.* **15** (1) (2023), 1–1.

[4] F. Etayo, A. de Francisco, R. Santamaría, Classification of almost Norden golden manifolds, *Bull. Malays. Math. Sci. Soc.* **43** (6) (2020), 3941–3961.

[5] F. Etayo, R. Santamaría, The well adapted connection of a $(J^2 = \pm 1)$-metric manifold, *RACSAM* **111** (2017), 355–375.

[6] G.T. Ganchev, A.V. Borisov, *Note on the almost complex manifolds with a Norden metric*, C. R. Acad. Bulgare Sci. **39** (5) (1986), 31–34.

[7] M. Gök, E. Kılıç, Non-invariant submanifolds of almost poly-Norden Riemannian manifolds, *J. Sci. Arts* **24** (3) (2024), 453–464.

[8] S. Y. Perktaş, Submanifolds of almost poly-Norden Riemannian manifolds, *Turk. J. Math.* **44** (1) (2020), 31–49.

[9] T. Acet, S. Yüksel Perktaş, E. Kılıç, On some types of light-like submanifolds of poly-Norden semi-Riemannian manifolds, *Filomat* **37** (10) (2023), 3225–3240.

[10] V. Ayhan, S. Yüksel Perktaş, Slant submanifolds of almost poly-Norden Riemannian manifolds, *Int. J. Maps Math.* **6** (1) (2023), 22–36.

On Hypersurfaces in the Minkowski Space $\mathbb{E}_1^5$

Zülal Derin Yaqub, Nurettin Cenk Turgay

Mathematics, Istanbul Technical University, Istanbul, Turkiye, derinzulal@gmail.com
Mathematics, Istanbul Technical University, Istanbul, Turkiye, turgayn@itu.edu.tr

Abstract

In this work, we investigate Lorentzian hypersurfaces in the Minkowski space $\mathbb{E}_1^5$ with the non-diagonalizable shape operator. By constructing a suitable pseudo-orthonormal frame, we study some of intrinsic and extrinsic properties of such hypersurfaces. Furthermore, by employing the Fundamental Theorem of Submanifolds, we establish local existence and uniqueness results for Lorentzian hypersurfaces admitting the prescribed non-diagonalizable shape operator. The obtained results contribute to the classification theory of Lorentzian hypersurfaces in Minkowski spaces and extend previous studies concerning hypersurfaces with non-diagonalizable shape operators.

Keywords: Minkowski space; Lorentzian hypersurfaces, non-diagona lizable shape operator, pseudo-orthonormal frame, existence and uniqueness.
2010 Mathematics Subject Classification: 53C42, 53B30, 53C50.
Acknowledgements: This work was carried out within the scope of a project supported by TÜBİTAK, the Scientific and Technological Research Council of Türkiye, under the BİDEB-2218 National Postdoctoral Research Fellowship Program (Project No. 123C433).

References

[1] A. Kayhan, N. C. Turgay, Biconservative hypersurfaces in E_1^4 with non-diagonalizable shape operator, *J. Geom. Phys.* **158** (2020), 103875.

[2] Y. Liang, C. Xia, hypersurfaces with non-diagonalizable shape operator, *Results in Mathematics* **74** (2019), 117.

[3] M. Dajczer, M. Antonucci, G. Oliveira, P. Lima-Filho, R. Tojeiro, Submanifolds and Isometric Immersions, *Mathematics Lecture Series*, Publish or Perish, Houston, Texas, 13, 1990.

22nd International Geometry Symposium
8 - 10 July 2026, Süleyman Demirel University, TÜRKİYE

On the Main Properties of the Fiberwise Berger Type Deformed Sasaki Metric on the Coframe Bundle

Habil Fattayev

Department Algebra and Geometry, Baku State University, Baku, Azerbaijan, e-mail: h-fattayev@mail.ru

Abstract

The study of the properties of geometric objects on the fiber bundles with the Sasaki metrics imposes flatness on the base manifold ([1], [2]). To overcome this obstacle, some authors have defined new modifications of the Sasaki metric in various bundles (see, for example, [3], [4]). The main goal of this talk is to define the fiberwise Berger type deformed Sasaki metric on the linear coframe bundle of a Riemannian manifold and study its main properties. Let (M, φ, g) be an almost anti-para-Hermitian manifold and $F^*(M)$ be its coframe bundle [4]. A fiberwise Berger type deformed Sasaki metric is a Riemannian metric ${}^{BS}g$ on the linear coframe bundle $F^*(M)$ such that

$$
\begin{aligned}
&{}^{BS}g({}^{H}X, {}^{H}Y) = {}^{V}(g(X,Y)) = g(X,Y),\\
&{}^{BS}g({}^{V_\alpha}\omega, {}^{H}Y) = 0,\\
&{}^{BS}g({}^{V_\alpha}\omega, {}^{V_\beta}\theta) = 0,\ \alpha \neq \beta,\\
&{}^{BS}g({}^{V_\alpha}\omega, {}^{V_\alpha}\theta) = g^{-1}(\omega,\theta) + \delta^2 g^{-1}(\omega, X^\alpha\varphi) g^{-1}(\theta, X^\alpha\varphi)
\end{aligned}
$$

for all $X, Y \in \Im_0^1(M)$ and $\omega, \theta \in \Im_1^0(M)$, where δ is some constant. The basic properties of the Levi-Civita connection ${}^{BS}\nabla$ of the fiberwise Berger type deformed Sasaki metric ${}^{BS}g$ are investigated, and the values of the components ${}^{BS}\Gamma_{IJ}^{K}$ of this connection for different indices are also found. The components of the Riemannian curvature tensor of coframe bundle $F^*(M)$ with the metric ${}^{BS}g$ are calculated.

Keywords: Coframe bundle, Berger type deformed Sasaki metric, Levi-Civita connection, curvature tensor.
2010 Mathematics Subject Classification: 53C12, 57R30.

References

[1] O. Kowalski, Curvature of the induced Riemannian metric of the tangent bundle of Riemannian manifold, *J. Reine Angew. Math.* **250** (1971), 124–129.

[2] E. Musso, F. Tricerri, Riemannian metrics on tangent bundles, *Ann. Mat. Pura. App.* **150**(4) (1988), 1–10.

[3] M. Altunbas, R. Simsek, A. Gezer, A study Concerning Berger type deformed Sasaki Metric on the Tangent Bundle, *Zh. Mat. Fiz. Anal. Geom.* **15**(4) (2019), 435–447.

[4] H. Fattayev, A. Salimov, Some properties of rescaled Sasaki type metric on the coframe bundle, *Turkish Journal of Mathematics* **47**(4) (2025), 518–534.

The Schouten-van Kampen Connection on Riemannian Concircular Structure Manifolds

Ayda Murat, Özge Özbek, Semra Zeren, Ahmet Yıldız
Department of Mathematics, İnönü University, Malatya, Türkiye, ayda.murat44@gmail.com
Department of Mathematics, Inonu University, Malatya, Türkiye, ozgeordek@gmail.com
Department of Fundamental Engineering Sciences, Sivas University of Science and Technology, Sivas, Türkiye, zerensemra@sivas.edu.tr
Department of Mathematics, İnönü University, Malatya, Türkiye, a.yildiz@inonu.edu.tr

Abstract

In this paper, we investigate some flatness and some symmetry properties on a Riemannian concircular structure manifold M with respect to the Schouten-van Kampen connection. Firstly, we prove an existence theorem for this type of connection on the manifold M. Then, we study conformally flat, projectively flat, and concircularly flat Riemannian concircular structure manifolds with respect to the Schouten-van Kampen connection. We also study semi-symmetric Riemannian concircular structure manifolds with respect to the Schouten-van Kampen connection.

Keywords: Riemannian concircular structure manifold, Schouten-van Kampen connection, semi-symmetric.
2010 Mathematics Subject Classification: 53B05, 53C15, 53C25.

References

[1] S. K. Chaubey, Y. J. Suh, Riemannian concircular structure manifolds, *Filomat* **36**(19) (2022), 6699–6711.

[2] B. Y. Chen, Some results on concircular vector fields and their applications to Ricci solitons, *Bull. Korean Math. Soc.* **52**(5) (2015), 1535–1547.

[3] B. Y. Chen, S. K. Chaubey, U. C. De, G. E. Vilcu, Almost Ricci solitons on Riemannian concircular structure 3-manifolds, *Ricerche di Matematica*, to appear, (2026).

[4] K. Yano, Concircular geometry I. Concircular transformations, *Proc. Imp. Acad. Tokyo* **16** (1940), 195–200.

[5] S. Zeren, A. Yıldız, On Trans-Sasakian manifolds with the Schouten-van Kampen connection, *Konuralp Journal of Mathematics* **8**(2) (2020), 152–157.

[6] A. Yıldız, f-Kenmotsu manifolds with the Schouten-van Kampen connection, *Institut Mathématique. Publications. Nouvelle Série*, **102**(116) (2017), 93–105.

[7] J. A. Schouten, Ricci-Calculus: An introduction to tensor analysis and its geometrical applications, 2nd. ed., Die Grundlehren der mathematischen Wissenschaften, Springer-Verlag, Berlin-Gottingen-Heidelberg, 1954.

[8] Z. Olszak. The Schouten-van Kampen affine connection adapted to an almost (para) contact metric structure, *Publ. Inst. Math. (Beograd) (N.S.)* **94**(108) (2013), 31–42.

[9] C. A. Mantica, L. G. Molinari, Generalized Robertson-Walker spacetimes, a survey, *Int. J. Geom. Methods Mod. Phys.* **14** (3) (2017), 1730001, 27.

[10] M. C. Chaki, R. K. Maity, On quasi Einstein manifolds, *Publ. Math. Debrecen* **57**(3-4) (2000), 297–306.

Tzitzeica Curves with Myller Configuration in 4D Euclidean Space

Zehra İşbilir, Bahar Doğan Yazıcı, Murat Tosun
Department of Mathematics, Düzce University, Düzce, Türkiye,
zehraisbilir@duzce.edu.tr
Department of Mathematics, Bilecik Şeyh Edebali University, Bilecik, Türkiye,
bahar.dogan@bilecik.edu.tr
Department of Mathematics, Sakarya University, Sakarya, Türkiye,
tosun@sakarya.edu.tr

Abstract

In this paper, our aim is to investigate a new curve family bringing together the concepts of Tzitzeica curves and Frenet-type frame with Myller configuration in 4D Euclidean space $\mathbb{E}_4$. For this aim, the newly constructed family of curves in this context constitutes a family of generalized Tzitzeica curves with the Frenet-type frame in Myller configuration for $\mathbb{E}_4$, which also covers several special cases, one of which corresponds to the classical Tzitzeica curves with the Frenet frame in $\mathbb{E}_4$, due to the inherent structures of the Frenet-type frame with Myller configuration in $\mathbb{E}_4$. Also, we derive some geometrical properties and relations with respect to the invariants and curvatures for Tzitzeica curves with Frenet-type frame for Myller configuration in $\mathbb{E}_4$.

Keywords: Tzitzeica curves, Myller configuration, Frenet-type frame, 4D Euclidean space.
2010 Mathematics Subject Classification: 53A04, 53A70.

References

[1] A. Alkan, M. Önder, Some special helices in Myller configuration, *Appl. Math. E-Notes* **26** (2026), 93–104.

[2] M. E. Aydın, M. Ergüt, Non-null curves of Tzitzeica type in Minkowski 3-space, *Romanian J. of Math. Comp. Sci.* **4**(1) (2014), 81–90.

[3] B. Bayram, E. Tunç, K. Arslan, G. Öztürk, On Tzitzeica curves in Euclidean 3-space $\mathbb{E}^3$, *Facta Univ., Math. Inform.* **33**(3) (2018), 409–416. https://doi.org/10.22190/FUMI1803409B

[4] N. Bila, Symmetry reductions for the Tzitzeica curve equation, *Math and Computer Science Faculty Working Paper*, **16** (2012), 11 pages.

[5] A. Bobe, W. G. Boskoff, M. G. Ciuca, Tzitzeica type centro-affine invariants in Minkowski space, *An. St. Univ. Ovidius Constanta, Ser. Mat.* **20**(2) (2012), 27–34. https://doi.org/10.2478/v10309-012-0037-0

[6] B. Bronson, G. Costa, Diferansiyel Denklemler, McGraw-Hill, Nobel Press (Hacısalihoğlu, H. H.), Third Edition, (In Turkish).

[7] B. Y. Chen, Constant ratio hypersurfaces, Soochow J. Math. **28** (2001), 353–362.

[8] B. Y. Chen, Geometry of warped products as Riemannian submanifolds and related problems, Soochow J. Math. **28**(2) (2002), 125–156.

[9] B. Y. Chen, Geometry of position function of totally real submanifolds in complex Euclidean spaces, Kragujev. J. Math. **37**(2) (2013), 201–215.

[10] M. Crăşmăreann, Cylindrical Tzitzeica curves implies forced harmonic oscillators, *Balk. J. Geom. Appl.*, **7**(1) (2002), 37–42.

[11] O. Constantinescu, M. Crăşmăreann, A new Tzitzeica hypersurface and cubic Finslerian metrics of Berwall type, *Balk. J. Geom. Appl.* 16(2) (2011), 27-–34.

[12] G. Darboux, Leçons Sur La Theorie Generale Des Surfaces, Vol. I, II, III, IV. Paris: Gauthier-Villars, 1896.

[13] B. Doğan Yazıcı, S. Özkaldı Karakuş, M. Tosun, On framed Tzitzeica curves in Euclidean space, *Facta Univ., Math. Inform.*, **37**(2) (2022), 307–319. https://doi.org/10.22190/FUMI211025021D

[14] K. Eren, S. Ersoy, Characterizations of Tzitzeica curves using Bishop frames, *Math. Meth. Appl. Sci.* **45**(18) (2022), 12046–12059. https://doi.org/10.1002/mma.7483

[15] F. Frenet, Sur les courbes á double courbure, *Journal de Mathématiques Pures et Appliquées* **17** (1852), 437–447.

[16] H. Gluck, Higher curvatures of curves in Euclidean space, *Amer. Math. Monthly* **73**(7) (1966), 699–704. https://doi.org/10.2307/2313974

[17] B. Heroiu, Versor fields along a curve in a four dimensional Lorentz space, *Journal of Advanced Mathematical Studies* **4**(1) (2011), 49–57.

[18] K. İlarslan, E. Nešović, Some characterizations of osculating curves in the Euclidean spaces, Demonstr. Math. **41**(4) (2008), 931–940.

[19] Z. İşbilir, M. Tosun, On generalized osculating-type curves in Myller configuration, *An. St. Univ. Ovidius Constanta, Ser. Mat.* **32**(2) (2024) 85–97. https://doi.org/10.2478/auom-2024-0020

[20] Z. İşbilir, M. Tosun, A new insight on rectifying-type curves in Euclidean 4-space, *Int. Electron. J. Geom.* **16**(2) (2023), 644–652. https://doi.org/10.36890/iejg.1291893

[21] Z. İşbilir, M. Tosun, Generalized Smarandache curves with Frenet-type frame, *Honam Math. J.* **46**(2) (2024), 181–197. https://doi.org/10.5831/HMJ.2024.46.2.181

[22] Z. İşbilir, B. Doğan Yazıcı, M. Tosun, On special curves in Lie groups with Myller configuration, *Math. Meth. Appl. Sci.* **47**(14) (2024), 11693–11708. https://doi.org/10.1002/mma.10149

[23] Z. İşbilir, B. Doğan Yazıcı, M. Tosun, On construction of Mannheim curves with Myller configuration, *Turk. J. Math.* (in press) (2026).

[24] Z. İşbilir, B. Doğan Yazıcı, M. Tosun, Generalized ruled surfaces in Myller configuration, *Filomat* **39**(15) (2025), 5253–5272. https://doi.org/10.2298/fil2515253i

[25] M. K. Karacan, B. Bükçü, On the elliptic cylindrical Tzitzeica curves in Minkowski 3-space, *Sci. Manga.* **5**(3) (2009), 44–48.

[26] M. K. Karacan, B. Bükçü, On the hyperbolic cylindrical Tzitzeica curves in Minkowski 3-space, *Journal of Balıkesir University Institute of Science and Technology* **10**(1) (2009), 46–51.

[27] W. Kuhnel, Differential Geometry: Curves-Surfaces-Manifolds, Braunschweig, Wiesbaden, 1999.

[28] Y. Li, A. Alkan, M. Önder, Y. Xie, Slant helices and Darboux helices in Myller configuration, *Axioms* **14**(5) (2025), 353, 13 pages. https://doi.org/10.3390/axioms14050353

[29] G. Macsim, A. Mihai, A. Olteanu, Special curves in a Myller configuration, Proceedings of the 16th Workshop on Mathematics, Computer Science and Technical Education, Department of Mathematics and Computer Science 2019, 2, 78–84.

[30] G. Macsim, A. Mihai, A. Olteanu, Curves in a Myller configuration. International Conference on Applied and Pure Mathematics (ICAPM 2017), Iaşi, 2–5 November, 2017.

[31] G. Macsim, A. Mihai, A. Olteanu, On rectifying-type curves in a Myller configuration, *Bull. Korean Math. Soc.* **56**(2) (2019), 383–390. https://doi.org/10.4134/BKMS.b180279

[32] R. Miron, The Geometry of Myller Configurations, Applications to Theory of Surfaces and Nonholonomic Manifolds. Romanian Academy, 2010.

[33] R. Miron, Geometria unor configuraţii Myller, *Analele Şt. Univ.* **VI**(3) (1960).

[34] M. Özdemir, Diferansiyel Geometri. Antalya: Altın Nokta, 2020.

[35] G. Öztürk, S. Gürpınar, K. Arslan, A new characterization of curves in Euclidean 4-space $\mathbb{E}^4$, Buletinul Academiei de Stiinte a Republicii Moldova. Matematica, **83**(1) (2017), 39–50.

[36] J. A. Serret, Sur quelques formules relatives á la théorie des courbes á double courbure, *Journal de Mathématiques Pures et Appliquées* **16** (1851), 193–207.

[37] E. Tunç, B. Bayram, A new characterization of Tzitzeica curves in Euclidean 4-space, *Fundamentals of Contemporary Mathematical Sciences* **4**(2) (2023), 77–86. https://doi.org/10.54974/fcmathsci.1176710

[38] E. Tunç, B. Bayram, A note on Tzitzeica curves in Euclidean 4-space $\mathbb{E}^4$, International Theory, Research and Reviews in Science and Mathematics, October 2023, Editor: H. Akgül, Serüven Yayınevi, Chapter 10, pp. 141–166.

[39] G. Tzitzeica, Sur une nouvelle classe de surfaces, *Comptes Rendus des Seances de l'Academie des Sciences Paris* **144**(1) (1907), 1257–1259.

[40] G. Tzitzeica, Sur certaines courbes gauches, *Annales scientifiques de l'École normale supérieure* **28**(3) (1911), 9–32.

[41] G. E. Vilcu, A geometric perspective on the generalized Cobb-Douglas production function, *Appl. Math. Lett.* **24**(5) (2011), 777–783. https://doi.org/10.1016/j.aml.2010.12.038

A Study on the Relationship Between Polyhedra and Minkowski Geometries

Özcan Gelişgen, Temel Ermiş

Department of Mathematics and Computer Sciences, Eskişehir Osmangazi University, Eskişehir, Türkiye, gelisgen@ogu.edu.tr

Department of Mathematics and Computer Sciences, Eskişehir Osmangazi University, Eskişehir, Türkiye, termis@ogu.edu.tr

Abstract

In this presentation, two remarkable and intriguing research areas within geometry—an intellectual discipline as ancient as human civilization itself—are examined: polyhedra and metric geometries. The first topic concerns three elegant and fascinating classes of polyhedra, namely the Platonic solids, the Archimedean solids and the Catalan solids, which are three-dimensional geometric objects that have attracted human interest for millennia. The second topic focuses on metric geometries, a field that is considerably more recent than the study of polyhedra, possessing a history of little more than a century. Recent years have witnessed an increasing number of studies situated at the intersection of these two areas. Motivated by this growing body of research, the present work investigates the existence of metrics—and consequently the corresponding metric geometries—under which the Platonic, Archimedean, and Catalan solids are regarded as spheres. Subsequently, several geometric properties of the resulting metric spaces are analyzed. Finally, in accordance with Felix Klein's conception of geometry as the study of properties that remain invariant under specific transformations of a set onto itself, the distance-preserving transformations (i.e., isometries) of the constructed metric spaces are examined and explicitly characterized. It is then demonstrated that the isometry group of the corresponding spaces is given by the semidirect product of the Euclidean translation group in three-dimensional analytic space and the symmetry group of the polyhedron under consideration.

Keywords: Platonic solids, Archimedean solids, Catalan solids.
2010 Mathematics Subject Classification: 51B20, 51F99, 51N25.

References

[1] A. G. Horvath, Semi-indefinite inner product and generalized Minkowski spaces, *Journal of Geometry and Physics* **60(9)** (2010), 1190-1208.

[2] A.C. Thompson, Minkowski Geometry, Cambridge University Press, Cambridge, 1996.

[3] H. S. M. Coxeter, Regular Polytopes, Pitman, 1947.

Cross Product and Slant Curves in 3-Dimensional Contact Metric Spaces

Çetin Camcı

Department of Mathematics, Çanakkale Onsekiz Mart University, Çanakkale, Türkiye,
camci@comu.edu.tr

Abstract

In three-dimensional contact manifolds, curves that make a constant angle with the tangent vector field and the Reeb vector field are called slant curves. In this study, we explain how the cross product provides operational convenience in three-dimensional contact manifolds, and we investigate the relationship between the characterization of slant curves and Lancret's theorem.

Keywords: Hyperbolic spinors, Mannheim curve, Minkowski 3-space.
2010 Mathematics Subject Classification: 15A66, 51B20.

References

[1] C. Baikoussis, D. E. Blair, On Legendre curves in contact 3-manifolds, *Geom. Dedicata* **49** (1994), 135-142.

[2] C. Camcı, Extended cross product in a 3-dimensionsal almost contact metric manifold with applications to curve theory, *Turk. J. Math.* **36** (2012) 305-318.

[3] J.-E. Lee, Slant curves and contact magnetic curves in Sasakian Lorentzian 3- manifolds, *Symmetry* **11**(6) (2019), 784.

[4] J. T. Cho, J.-I. Inoguchi, J.-E. Lee, On slant curves in Sasakian 3-manifolds, *Bull. Austral. Math. Soc.* **74** (2006), 359-367.

[5] J.-I. Inoguchi, M. I. Munteanu, Slant curves and magnetic curves, Contact Geometry of Slant Submanifolds, Springer Nature Singapore, Singapore, (2022), pp. 199-259.

[6] J.-I. Inoguchi, J.-E. Lee, Slant curves in 3-dimensional almost contact metric geometry, *International Electronic Journal of Geometry* **8** (2015), 106-146.

22nd International Geometry Symposium
8 - 10 July 2026, Süleyman Demirel University, TÜRKİYE

Hyperbolic Spinor Equations of Mannheim Curves in Minkowski 3-Space

Zeynep İsabeyoğlu, Tülay Erişir

Department of Mathematics, Erzincan Binali Yıldırım University, Erzincan, Türkiye, zeyneposkay88@gmail.com

Department of Mathematics, Erzincan Binali Yıldırım University, Erzincan, Türkiye, tulay.erisir@erzincan.edu.tr

Abstract

Spinors have been used as vector-like structures in physics until now. When the spin matrices were introduced by Pauli in 1927, the spinors were adapted to mathematical physics [1]. Later, the relationships between spinors and the Lorentz group were demonstrated by Dirac [2]. Spinors are complex column vectors and move on Pauli spin matrices. The French mathematician Cartan, one of the pioneers in studying Lie groups, was the first person to geometrically examine spinors [3]. Cartan obtained the spinor formulas of the basic definitions in geometry and emphasized that isotropic vectors in C^3 create a surface with two-dimensional in complex space C^2. Moreover, Cartan expressed that these complex vectors in the space C^2 are spinors [2]. In another study, Torres del Castillo and Barrales demonstrated the theory of curves in 3-dimensional Euclidean space using spinors [4]. Then, a lot of studies about spinors and curve theory in Euclidean and Minkowski space [4–16]. This paper is devoted to the hyperbolic spinor representation of timelike Mannheim curve pairs in Minkowski 3-space. Starting from a Mannheim curve pair, we assign two hyperbolic spinors corresponding to the timelike Mannheim partner curves and describe their relations through the Frenet frames. By making use of the intrinsic connection between Mannheim curve pair, we obtain explicit relations among the associated hyperbolic spinors and present the corresponding hyperbolic spinor equations.

Keywords: Hyperbolic spinors, Mannheim curve, Minkowski 3-space.
2010 Mathematics Subject Classification: 15A66, 51B20.

References

[1] W. Pauli, Zur quantenmechanik des magnetischen elektrons, *Zeitschrift für Physik* **43** (1927), 601–623.

[2] P. A. M. Dirac, The quantum theory of the electron, *Proceeding of the Royal Society A.* **117**(778) (1928), 610–624.

[3] E. Cartan, The Theory of Spinors, Dover Publications, New York, 1966.

[4] G. F. Torres del Castillo, G. S. Barrales, Spinor formulation of the differential geometry of curves, *Revista Colombiana de Matematicas* **38** (2004), 27–34.

[5] D. Ünal, İ. Kişi, M. Tosun, Spinor Bishop equations of curves in Euclidean 3-space, *Adv. Appl. Cliff. Algebr.* **23**(3) (2013), 757-765.

[6] İ. Kişi, M. Tosun, Spinor Darboux equations of curves in Euclidean 3-space, *Mathematica Moravica* **19**(2) (2015), 87–93.

[7] M. Tarakçıoğlu, T. Erişir, M. A. Güngör, M. Tosun, The hyperbolic spinor representation of transformations in R_1^3 by means of split quaternions, *Adv. Appl. Cliff. Algebr.* **28**(1) (2018), 26.

[8] O. Z. Okuyucu, Ö. G. Yıldız, M. Tosun, Spinor Frenet equations in three dimensional Lie Groups, *Adv. Appl. Cliff. Algebr.* **26** (2016) 1341–1348.

[9] S. Şenyurt, A. Çalışkan, Spinor formulation of Sabban frame of curve on S^2, *Pure Mathematical Sciences* **4**(1) (2015), 37–42.

[10] T. Erişir, M. A. Güngör, M. Tosun, Geometry of the hyperbolic spinors corresponding to alternative frame, *Adv. Appl. Cliff. Algebr.* **25**(4) (2015), 799–810.

[11] T. Erişir, N. C. Kardağ, Spinor representations of involute evolute curves in E^3, *Fundam. J. Math. Appl.* **2**(2) (2019), 148–155.

[12] T. Erişir, On spinor construction of Bertrand curves, *AIMS Mathematics*, **6**(4) (2021), 3583–3591.

[13] T. Erişir, H. Köse Öztaş, Spinor equations of successor curves, *Univ. J. Math. Appl.* **5**(1) (2022), 32–41.

[14] T. Erişir, Z. İsabeyoğlu, The spinor expressions of Mannheim curves in Euclidean 3-space, *Int. Electron. J. Geom.* **16**(1) (2023), 111–121.

[15] Y. Balcı, T. Erişir, M. A. Güngör, Hyperbolic spinor Darboux equations of spacelike curves in Minkowski 3-spacelike, *Journal of the Chungcheong Mathematical Society* **28**(4) (2015), 525–535.

[16] Z. Ketenci, T. Erişir, M. A. Güngör, A construction of hyperbolic spinors according to Frenet frame in Minkowski space, *Journal of Dynamical Systems and Geometric Theories* **13**(2) (2015), 179–193.

On the Geometric Meaning of the Rotation Angle Around a Lightlike Axis in Minkowski-3 Space

Mehmet Duru, Harun Barış Çolakoğlu
Department of Mathematics, Akdeniz University, Antalya, Türkiye,
mehmetduruedu@gmail.com
Department of Computer Technologies, Akdeniz University, Antalya, Turkiye,
hbcolakoglu@akdeniz.edu.tr

Abstract

In this study, we consider the Lorentzian rotation around a lightlike axis. First, the relationship between the Cartan frame and the rotation angle with a lightlike axis is given. Then, we present a geometric characterization for the rotation angle between two vectors that can be rotated into each other under a Lorentzian rotation around a lightlike axis. Finally, we provide a definition for the measurement of the angle between two spacelike vectors whose vector product is lightlike.

Keywords: Lorentzian scalar product, Lorentzian vector product, Lorentzian rotation matrix, Cartan frame, lightlike axis.
2010 Mathematics Subject Classification: 53B30, 15A63, 53A35, 70B10.

References

[1] H. B. Çolakoğlu, M. Duru, A. Yılmaz Ceylan, Parabolic and linear rotational motions on cones and hyperboloids, *Axioms* **14**(8) (2025), 612.

[2] E. Nešovic, On rotation about lightlike axis in three-dimensional Minkowski space, *Adv. Appl. Clifford Algebras* **26** (1) (2016), 237–251.

[3] J. G. Ratcliffe, Foundations of Hyperbolic Manifolds. Springer, Switzerland, 2006.

[4] B. W. O'Neill, Semi-Riemannian Geometry with Applications to Relativity, Academic Press, California, USA, 1983.

[5] W. Sodsiri, Lorentzian motions in Minkowski 3-space, KKU. Sci. J. **34**(3) (2006), 242–249.

[6] R. López, Differential geometry of curves and surfaces in Lorentz-Minkowski space, *International Electronic Journal of Geometry* **7** (2014), 44–107.

New Classes of Supervectors

Dilek Demirhan, Cansel Aycan
Faculty of Sciences, Department of Mathematics, Pamukkale University, Denizli, Türkiye, dilekdemirhan11@gmail.com
Faculty of Sciences, Department of Mathematics, Pamukkale University, Denizli, Türkiye, c_aycan@pau.edu.tr

Abstract

In recent years, many studies have been conducted in the field of differential geometry using supervectors on superspace. This study presents a restructuring and decomposition of the even component, alongside a unification of the odd and body components, thereby contributing seven new types of supervectors over the same class of Grassmann variables to the literature in the $\mathbb{R}^{m_{\text{body}}|n_{\text{even}},n_{\text{odd}}}$ superspace.

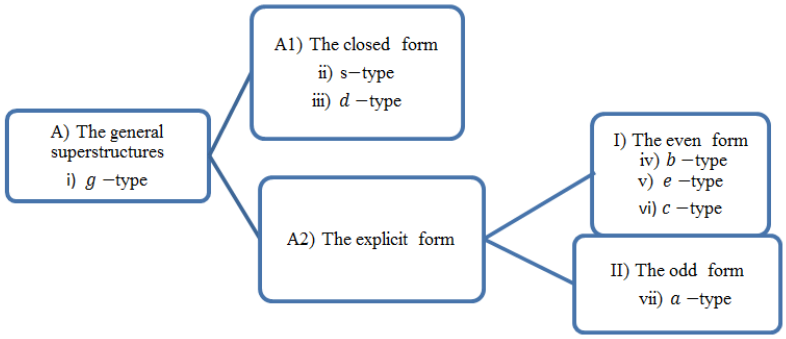

Figure 1: Classification and hierarchical decomposition of the general superstructures.

Keywords: Grassmann algebra, supervector, superspace.
2010 Mathematics Subject Classification: 17A70, 15A75, 58A50, 81T60.

References

[1] A. Rogers, Supermanifolds: Theory and Applications. Singapore, World Scientific Publ., 2007.

[2] I.L. Buchbinder, S. M. Kuzenko, Ideas and Methods of Supersymmetry and Supergravity or a Walk Through Superspace, Russia: Taylor & Francis Group SCOAP 3, 2021.

[3] F. A. Berezin, Introduction to Superanalysis, III. Series, Holland: D. Reidel Pub., 1987.

A Characterization of Constant Mean Curvature Revolution Surfaces in Galilean Space

İsmet Gölgeleyen

Department of Mathematics, Faculty of Science, Zonguldak Bülent Ecevit University, Zonguldak 67100, Türkiye, ismet.golgeleyen@beun.edu.tr

Abstract

The study of surfaces with prescribed curvature properties constitutes an important topic in differential geometry. Among these, surfaces of revolution provide a natural class of examples due to their geometric simplicity and rich structure. While minimal surfaces of revolution in Galilean geometry have attracted considerable attention, the case of non-zero constant mean curvature has not yet been systematically studied. In this talk, we investigate revolution surfaces in Galilean 3-space whose mean curvature is a non-zero constant. By deriving and analyzing the governing differential equations, we obtain a characterization of such surfaces and establish explicit parametrizations for the resulting families. The obtained results provide new examples of constant mean curvature surfaces in Galilean geometry and highlight several geometric properties that distinguish them from minimal surfaces. Illustrative examples are presented to demonstrate the theoretical findings.

Keywords: Surfaces of revolution, mean curvature, Galilean geometry.
2010 Mathematics Subject Classification: 53A35, 53A10.

References

[1] B. Divjak, Ž. Milin-Šipuš, Special curves on ruled surfaces in Galilean and pseudo-Galilean spaces, *Acta Math. Hung.* **98** (2003), 203–215.

[2] M. Dede, C. Ekici, W. Goemans, Surfaces of revolution with vanishing curvature in the Galilean 3-space, *J. Math. Phys. Anal. Geom.* **14** (2018), 141–152.

[3] İ. Gölgeleyen, Y. Yaylı, E. Y. Bulgan, Developable ruled surfaces with constant mean curvature along a curve, *Mathematics* **14** (2026), 234.

Geometric Invariants and Higher-Order Variations of Paths in Lorentzian 3-Space

Fatma Almaz

Department of Mathematics, Faculty of Arts and Sciences, Batman University, Batman, Türkiye, fatma.almaz@batman.edu.tr

Abstract

This paper explores the intrinsic and extrinsic differential behavior of structural curves embedded within three-dimensional Minkowski backgrounds. By utilizing an adaptive localized framing technique on timelike surfaces, the third-order evolution equations of parametrized trajectories are formalized, isolating the geometric contributions to non-uniform path transitions. Furthermore, a specialized decomposition for the underlying equations of motion is established, revealing novel conserved quantities and structural symmetries that govern the space. The resulting formulation offers an alternative geometric perspective on the evolution of constraints in non-Euclidean spaces, providing a distinct framework from traditional kinematic descriptions.

Keywords: Higher-order differential invariants, geometric trajectories, conserved quantities.
2010 Mathematics Subject Classification: 37K05, 53A35.

References

[1] J. Casey, Siacci's resolution of the acceleration vector for a space curve, *Meccanica* **46**(2) (2011), 471-476.

[2] M. D. Carmo, Differential Geometry of Curves Surface, Prentice-Hall, Englewood Cliffs, NJ, 1976.

[3] J. H. Bickford, Mechanisms for Intermittent Motion. New York: Industrial Press, 1972.

[4] A.A. Ergin, Timelike Darboux curves on a timelike surface $M \subset E_1^3$, *Hadronic Journal* (2001), 701-712.

[5] F. Freudenstein, G. N. Sandor, Mechanical Design and Systems Handbook, New York: McGraw-Hill Companies, 1996.

[6] K. E. Özen, M. Güner, M. Tosun, A note on the acceleration and jerk in motion along a space curve, *An. St. Univ. Ovidius Constanta* **28**(1) (2020), 151-164.

[7] F. Siacci, Moto per una linea gobba, *Atti R Accad Sci. Torino* **14** (1879), 946-951.

[8] S. H. Schot, Jerk: the time rate of change of acceleration, *Am J Physiol* **46**(11) (1978), 1090-1094.

[9] T. Shifrin, Differential Geometry: A First Course in Curves and Surfaces. Athens: University of Georgia Press, 2015.

On Non-Lightlike Framed Immersions Under Rotations and Reflections

Gülsümnur Coşkun, Gülşah Aydın Şekerci

Department of Mathematics, Süleyman Demirel University, Isparta, Türkiye, gulsumnurcoskun@gmail.com

Department of Mathematics, Süleyman Demirel University, Isparta, Türkiye, gulsahaydin@sdu.edu.tr

Abstract

Framed immersions provide a useful notion for studying both regular and singular non-lightlike curves in Minkowski 3-space. In this context, we investigate non-lightlike framed immersions induced by rotated and reflected frame fields in Minkowski 3-space. We derive explicit relations between the curvature functions of the original and transformed framed curves. Using these relations, we examine the behavior of some geometric structures and show that several fundamental properties remain invariant under these transformations.

Keywords: Minkowski space, framed immersion, curvature, singular curves.
2010 Mathematics Subject Classification: 53A04, 57R45.

References

[1] J. W. Bruce, P. J. Giblin, Curves and Singularities, A Geometrical Introduction to Singularity Theory, Cambridge University Press, Cambridge, 1984.

[2] J. Huang, K. Yao, D. Pei, Non-lightlike framed rectifying curves in Minkowski 3-space, *Electron. Res. Arch.* **32** (2024), 5914–5925.

[3] S. Honda, M. Takahashi, Evolutes and focal surfaces of framed immersions in the Euclidean space, *Proc. Roy. Soc. Edinburgh Sect. A* **150** (2020), 497-516.

Spherical Loxodrome and Its Applications

Melike Bozkurt, Cansel Balıkçı, Murat Babaarslan

Department of Mathematics, Yozgat Bozok University, Yozgat, Türkiye, melikebozkurt1501@gmail.com
Department of Mathematics, Yozgat Bozok University, Yozgat, Türkiye, balikcicansel710@gmail.com
Department of Mathematics, Yozgat Bozok University, Yozgat, Türkiye, murat.babaarslan@bozok.edu.tr

Abstract

A spherical loxodrome (also known as a rhumb line) is a curve on a sphere that intersects all meridians at a constant angle. Instead of following the shortest path (a great circle (geodesic curve)), it spirals continuously toward the poles. The stereographic projections of loxodromes correspond to logarithmic spirals, while Mercator projections correspond to straight lines. Loxodromes are useful for marine and air navigation. In this talk, the Sabban frame of a loxodrome will be constructed, and its geodesic curvature will be found. Thus, the conditions for a loxodrome to be a geodesic curve will be investigated. Furthermore, the spherical evolute of the loxodrome will be calculated, and it will be investigated whether the spherical evolute is a rectifying curve. Finally, Bertrand pairs of the loxodrome and its special cases (e.g., the loxodrome being a circle or a geodesic curve) will be studied.

Keywords: Loxodrome, geodesic curve, spherical evolute, rectifying curve, Bertrand curve, Sabban frame.
2010 Mathematics Subject Classification: 53A04, 53A05.
Acknowledgements This study is supported by the Scientific Research Projects Unit of Yozgat Bozok University (Grant No: FLO-2025–1674).

References

[1] F. Kahraman Aksoyak, B. Bektaş Demirci, M. Babaarslan, Characterizations of loxodromes on rotational surfaces in Euclidean 3-space, *Int. Electron. J. Geom.* **16** (2023), 147–159.

[2] S. Izumiya, N. Takeuchi, Generic properties of helices and Bertrand curves, *J. Geom.* **74** (2002), 97–109.

[3] J. Koenderink, Solid Shape, MIT Press, Cambridge, 1990.

[4] S. Kos, D. Vranic, D. Zec, Differential equation of a loxodrome on a sphere, *J. Navig.* **52** (1999), 418–420.

[5] C. A. Noble, Note on loxodromes, *Bull. Amer. Math. Soc.* **12** (1905), 116–119.

The Characterizations of Timelike Loxodromes on Timelike Rotational Surfaces with Lightlike Axis in Minkowski 3-Space

Ferdağ Kahraman Aksoyak, Burcu Bektaş Demirci, Murat Babaarslan

Department of Elementary Mathematics Education, Kırşehir Ahi Evran University, Kırşehir, Türkiye, ferdag.aksoyak@ahievran.edu.tr
Department of Software Engineering, Fatih Sultan Mehmet Vakıf University, İstanbul, Türkiye, bbektas@fsm.edu.tr
Department of Mathematics, Yozgat Bozok University, Yozgat, Türkiye, murat.babaarslan@bozok.edu.tr

Abstract

In this paper, first we calculate the curvatures of timelike loxodromes on timelike rotational surfaces with lightlike axis. For some special timelike rotational surfaces such as flat, constant Gaussian curvature, minimal and Weingarten, we obtain the parametrizations of timelike loxodromes. Also, we prove that timelike loxodromes on timelike rotational surfaces are general helices whenever the surface is minimal, flat, or has a constant ratio of principal curvatures.

Keywords: Loxodrome, curvatures, Minkowski space.
2010 Mathematics Subject Classification: 53A35.
Acknowledgments: This work is supported by FSMVU Research Project Unit under grant number 2025B1Ç06D.

References

[1] J. Alexander, Loxodromes: A rhumb way to go, *Math. Mag.* **77** (2004), 349–356.

[2] M. Babaarslan, M. I. Munteanu, Timelike loxodromes on rotational surfaces in Minkowski 3-space *An. Stiint. Univ. Al. I. Cuza Iasi, Ser. Noua, Mat.* **61** (2015), 471–484.

[3] F. Kahraman Aksoyak, B. Bektaş Demirci, M. Babaarslan, Characterizations of loxodromes on rotational surfaces in Euclidean 3-space *Int. Electron. J. Geom.* **16** (2023) 147–159.

[4] C. G. Kendall, A study of the loxodrome on a general surface of revolution with special application to the conical spiral, Master dissertation, University of California, 1920.

[5] S. Kos, R. Filjar, M. Hess, Differential equation of the loxodrome on a rotational surface, In: International Technical Meeting of The Institute of Navigation, Anaheim, January 26–28, 2009, pp. 958–960.

[6] H. Leitão, J. A. Gaspar, Globes, rhumb tables, and the pre-history of the Mercator projection, *Imago Mundi.* **66** (2014), 180–195.

A Class of Framed Curves whose Darboux Vector is Perpendicular to a Constant Direction

Onur Kaya

Manisa Celal Bayar University, Department of Mathematics, 45140, Manisa, Türkiye, onur.kaya@cbu.edu.tr

Abstract

In this study, we investigate a special class of framed curves in the three-dimensional Euclidean space whose Darboux vector remains perpendicular to a fixed direction. This condition leads to a system of differential equations relating the curvature functions, which is shown to reduce to a family of exponential solutions. Using these results, we derive explicit forms of the associated moving frame vectors by solving the corresponding linear differential systems. Furthermore, we obtain the position vector of the framed curve through integration and analyze the structure of the fixed direction under the imposed constraint. The problem is further reduced to a second-order linear ordinary differential equation, whose general solution is determined. Finally, illustrative examples are presented to demonstrate the geometric behavior of this class of framed curves and to highlight the theoretical findings.

Keywords: Framed curve, Darboux vector, Euclidean space, curvature functions, differential equations, moving frame.
2010 Mathematics Subject Classification: 34A05, 34A30, 53A04.

References

[1] D. J. Struik, Lectures on Classical Differential Geometry: 2nd Edition, Dover Publications, New York, 1988.

[2] S. Honda, M. Takahashi, Framed curves in the Euclidean space, *Advances in Geometry* **16**(3) (2016), 265–276.

[3] A. Yücesan, G. Ö. Tükel, On the Darboux type vector of framed base curves, Karadeniz 13th International Conference on Applied Sciences (2023), 183–189.

A Tight Frame Approach to Sparse Image Representation: Comparing Discrete Fourier and Discrete Cosine Transforms

Gülden Kapusuz

Department of Basic Sciences, Isparta University of Applied Sciences, Isparta, Türkiye
guldenkapusuz92@gmail.com

Abstract

This study investigates the tight frame properties of the Discrete Fourier Transform and the Discrete Cosine Transform for the sparse representation of image signals. Grounded in frame theory, we mathematically analyze norm equivalence, energy conservation, and perfect reconstruction conditions. An algorithm is developed to comparatively evaluate the energy compaction, sparsity levels, and reconstruction performance of both transforms under coefficient thresholding. Findings demonstrate that the Discrete Cosine Transform outperforms the Discrete Fourier Transform in sparsification due to its real-based structure and superior low-frequency energy clustering. Furthermore, the mathematical stability of the tight frame architecture successfully controls information loss during coefficient elimination. This research provides an analytical contribution to sparsification approaches by validating the theoretical advantages of tight frames through concrete image processing applications.

Keywords: Tight frames, sparse representation.
2010 Mathematics Subject Classification: 42C15, 65T50, 94A08.

References

[1] R. C. Gonzalez, R. E. Woods, Digital Image Processing, 4th ed., Pearson, 2018.

[2] S. Mallat, A Wavelet Tour of Signal Processing: The Sparse Way, 3rd ed., Academic Press, 2008.

[3] R. J. Duffin, A. C. Schaeffer, A class of nonharmonic Fourier series, *Transactions of the American Mathematical Society* **72** (1952), 341–366.

[4] G. Kapusuz, Tight Frame ile l_p $(0 < p \leq 1)$ Minimizasyon Problemine Yeni Bir Yaklaşım ve Görüntü İşleme Uygulaması, Ph.D. thesis, Süleyman Demirel University, Isparta, 2025.

On Quasi Focal Curves of Non-null Curves with Quasi-frame

Gamze Tarım Kandilci, Başak Yağbasan, Cumali Ekici

Information Technology Department, Eskişehir Osmangazi University, Eskişehir, Türkiye, gamze.kandilci@ogu.edu.tr

Mathematics and Computer Science, Eskişehir Osmangazi University, Eskişehir, Türkiye, bskyagbasan@gmail.com

Mathematics and Computer Science, Eskişehir Osmangazi University, Eskişehir, Türkiye, cekici@ogu.edu.tr

Abstract

This study primarily characterizes focal curves using quasi-frame for non-null curves. The focal curve is expressed analytically through quasi-normal and quasi-binormal vectors. Accordingly, relationships have been found between the focal curvatures of the focal curve and the quasi-curvatures of non-null curves. Finally, some results are given regarding the case where non-null curves have constant quasi-curvatures for focal curvatures.

Keywords: Quasi-frame, Focal curve, Non-null curve.
2010 Mathematics Subject Classification: 53A35, 53B30.

References

[1] A. Yıldırım, Tubular surfaces aqccording to a focal curve in E^3, Turk. J. Math. Comput. Sci., **15**(2) (2023), 433–442.

[2] B. O'Neill, Semi-Riemannian Geometry with Applications to Relativity. Academic Press, 1983.

[3] C. Ekici, H. Tozak, M. Dede, Timelike directional tubular surfaces, Journal of Mathematical Analysis, **8**(5) (2017) 1–11.

[4] G. H. Georgiev, C. L. Dinkova, Generalized focal curves of Frenet curves in three-dimensional Euclidean space, Glob. J. Pure Appl. Math., **16**(6) (2020), 891–913.

[5] G. Öztürk, K. Arslan, On focal curves in Euclidean n-space R^n, Novi Sad Journal of Mathematics, **46**(1) (2010), 35–44.

[6] G. Tarım Kandilci, C. Ekici, M. Dede, Directional Bertrand curves in Minkowski space, Innovative Research in Natural Science and Mathematics, (2023), 15–38.

[7] M. Dede, C. Ekici, A. Görgülü, Directional q-frame along a space curve, IJARCSSE, **5** (2015), 775–780.

[8] M. Dede, C. Ekici, H. Tozak, Directional tubular surfaces, International Journal of Algebra, **9**(12) (2015), 527–535.

[9] R. Uribe-Vargas: On vertices, focal curvatures and differential geometry of space curves, Bull. Brazilian Math. Soc. **36**(3) (2005), 285–307.

[10] T. Körpınar, On quasi focal curves with quasi frame in space, Bol. Soc. Paran. Mat., **41** (2023), 1–3.

A Study on h-Almost Yamabe Solitons Along Riemannian Maps

Mehmet Çaldır, Nergiz (Önen) Poyraz, Erol Yaşar
Department of Mathematics, Çukurova University, Adana, Türkiye, mehmettcaldir@gmail.com
Department of Mathematics, Çukurova University, Adana, Türkiye, nonen@cu.edu.tr
Department of Mathematics, Mersin University, Mersin, Türkiye, yerol@mersin.edu.tr

Abstract

In this paper, we study Riemannian maps whose total manifolds or base manifolds admit a h-almost Yamabe soliton. We also obtain necessary conditions for any fiber of such Riemannian maps, the range space and its orthogonal complement spaces of such Riemannian maps to be h-almost Yamabe soliton. We also give a non-trivial example of such Riemannian maps.

Keywords: Riemannian maps, Yamabe soliton, scalar curvature.
2010 Mathematics Subject Classification: 53B20, 53C25, 53C43.

References

[1] R. Hamilton, The Ricci flow on surfaces, *Contemp. Math.* **71** (1988), 237–261.

[2] R. Hamilton, *Lectures on geometric flows*. Unpublished (1989).

[3] L. Ma, *Global Yamabe flow on asymptotically flat manifolds*, *J. Funct. Anal.* **281** (2021), Article 109229.

[4] L. Ma, Y. An, *The maximum principle and the Yamabe flow*. In: Chen, H., Rodino, L. (eds.) Partial Differential Equations and Their Applications (Wuhan, 1999), pp. 211-224, World Sci. River Edge.

[5] M.D. Siddiqi, F. Mofarreh, M. A. Akyol, A. H. Hakami, η-Ricci-Yamabe solitons along Riemannian submersions, *Axioms* **12**(8) (2023), 796.

[6] B. Şahin, Riemannian Submersions, Riemannian Maps in Hermitian Geometry, and Their Applications, Elsevier, Amsterdam, 2017.

[7] B. Şahin, Notes on Riemannian maps, *Sci. Bull. "Politeh." Univ. Buchar., Ser. A* **79**(3) (2017), 131–138.

[8] B. O'Neill, The fundamental equations of a submersion, *Mich. Math. J.* **13** (1966), 458–469.

[9] S. E. Meriç, Some remarks on Riemannian submersions admitting an almost Yamabe soliton, *Adiyaman Univ. J. Sci.* **10**(1) (2020), 295–306.

[10] A. E. Fischer, Riemannian maps between Riemannian manifolds, *Contemp. Math.* **132** (1992), 331-336.

[11] M. Falcitelli, S. Ianus and A. M. Pastore, Riemannian submersions and Related Topics World Scientific, Singapore, (2004).

[12] A. Yadav and K. Meena, Riemannian maps whose total manifolds admit a Ricci soliton, *J. Geom. Phys.*, **168** (2021), 1-13.

[13] A. Yadav and K. Meena, Clairaut Riemannian maps whose total manifolds admit a Ricci soliton, *Int. J. Geom. Methods Mod. Phys.*, **19**(2) (2022), article 2250024.

[14] A. Yadav and K. Meena, Riemannian maps whose base manifolds admit a Ricci soliton, *Publ. Math. Debrecen*, **103**(1-2) (2023), 115-139.

[15] S. E. Meriç, E. Kılıç, Riemannian submersions whose total manifolds admit a Ricci soliton, *Int. J. Geom. Methods Mod. Phys.*, **16** (2019) 1950196.

Minimisation of Curvature Energy in Parametric Curves: A Comparative Analysis of Classical and q-Gradient Descent Algorithms

Elvan Korkmaz, Serdar Soylu
Faculty of Arts and Science, Giresun University, Giresun, Türkiye,
elvan.korkmaz@giresun.edu.tr
Faculty of Arts and Science, Giresun University, Giresun, Türkiye,
serdar.soylu@giresun.edu.tr

Abstract

The geometric optimisation of parametric curves, particularly the minimisation of Euler elastica-based curvature energy ($\int \kappa^2 ds$), is a fundamental problem with critical applications in computer-aided geometric design (CAGD). However, the parameter spaces of such energy functionals inherently exhibit highly non-linear and multi-modal topologies. Classical gradient descent algorithms, relying strictly on infinitesimal local tangents, suffer from severe premature stagnation as they may become trapped in local minimum basins. To overcome these geometric limitations, this paper proposes a non-local optimisation strategy based on the Jackson q-derivative. To rigorously evaluate the algorithmic robustness and dimensional flexibility of this approach, comprehensive computational benchmarks were conducted on two distinct topologies: a 2D planar epicycloid with rigid topological constraints, and a 3D spatial circular helix featuring complex curvature-torsion coupling. The numerical experiments indicate that the classical method performs markedly worse than the q-gradient approach on the tested examples. In stark contrast, the explorative q-gradient regime ($q = 0.8$) successfully executed macroscopic secant tunnelling, achieving a substantial 93.3% energy reduction in the 2D plane and showed consistent performance, with a 32.1% reduction within the 3D spatial geometry. These results suggest that the q-parameter is a powerful dynamic control variable, positioning the q-gradient framework as an effective alternative for complex elastica optimisation.

Keywords: Geometric optimisation, curvature energy, q-calculus, q-gradient descent
2010 Mathematics Subject Classification: 49M37, 53A04.

References

[1] B. Audoly, Y. Pomeau, *Elasticity and Geometry: From Hair Curls to the Nonlinear Response of Shells*, Oxford University Press, 2010.

[2] T. Miura, K. Yoshizawa, General rigidity principles for stable and minimal elastic curves, *Journal für die reine und angewandte Mathematik (Crelle's Journal)*, **810** (2024), 253–281.

A Direct Proof of the Steiner-Lehmus Theorem Using GeoGebra

Recep Aslaner, İrem Üstüner

Institute of Education, İnönü University, Malatya, Türkiye, recep.aslaner@inonu.edu.tr
Institute of Education, İnönü University, Malatya, Türkiye,
37251405001@ogr.inonu.edu.tr

Abstract

This study examines an intriguing proposition from Euclidean geometry known in the literature as the Steiner–Lehmus Theorem. What makes this theorem particularly interesting is that, despite its simple statement—"If two angle bisectors of a triangle are equal in length, then the triangle is isosceles"—its proof has traditionally relied on indirect methods rather than a direct proof. In this study, the proposition is demonstrated using a direct proof approach with the aid of GeoGebra, a dynamic geometry software program. Furthermore, analogous statements involving other auxiliary elements of a triangle, namely medians and altitudes, are presented and shown to be valid. As a result, the relationships among these auxiliary elements in isosceles triangles are established.

Keywords: Euclidean geometry, isosceles triangle, angle bisector.
2010 Mathematics Subject Classification: 51M04.

References

[1] R. Aslaner, Dynamic Geometry Teaching. Anı Publishing, 2018.

[2] H. S. M. Coxeter, S. L. Greitzer, Geometry Revisited. Mathematical Association of America, 1967.

[3] R. Hammack, Book of Proof (3rd ed.). VCU Mathematics Textbook Series. https://people.clas.ufl.edu/pilyugin/files/Book-of-Proof.pdf (2018).

[4] G. Hine, N. McNab, Mathematics specialist: Year 11 ATAR course-Units 1, 2 (Australian curriculum). Academic Associates, (2014).

[5] L. Sauvé, The Steiner-Lehmus theorem, *Crux Mathematicorum* **2**(2) (1976), 19–21.

Double-Normal Chords and Singularities in a Hyperbolic Holditch-Type Problem

Serdar Soylu

Department of Mathematics, Giresun University, Giresun, Türkiye,
serdar.soylu@giresun.edu.tr

Abstract

A hyperbolic Holditch-type locus is considered for geodesic chords of fixed length on a strictly hyperbolically convex closed curve. The problem is formulated in the hyperboloid model, where the fixed-length condition becomes a Lorentzian constraint on the two endpoint parameters. The aim is to describe the local behavior of the locus traced by a point dividing each chord in a fixed ratio. Double-normal chords, where the connecting geodesic is orthogonal to the curve at both endpoints, form the critical configurations of the problem. Near such chords, the Hessian of the constraint determines the local structure. Its discriminant separates the possible regimes: the fixed-length chord locus may be locally isolated, split into two branches, develop cusp-type behavior, or exhibit endpoint-retrograde motion. The degenerate case is interpreted as a transition regime and is illustrated by the geodesic circle diameter model and small radial perturbations.

Keywords: Hyperbolic plane, Holditch locus, double-normal chord, ordinary cusp, retrograde motion.
Mathematics Subject Classification 2020: 52A55, 53A35, 53C22, 58K05.

References

[1] A. Broman, Holditch's theorem: a fresh look at a long-forgotten theorem, *Math. Mag.* **54** (1981), 99–108.

[2] R. Giambò, F. Giannoni, P. Piccione, Existence of orthogonal geodesic chords, *Nonlinear Analysis* **73** (2010), 290–337.

[3] J. Monterde, D. Rochera, On moving chords in constant curvature 2-manifolds, *Journal of Convex Analysis* **27** (2020), 1137–1156.

[4] J. W. Bruce, P. J. Giblin, *Curves and Singularities*, 2nd ed., Cambridge University Press, 1992.

[5] Á. G. Horváth, Diameter, width and thickness in the hyperbolic plane, *Journal of Geometry* **112** (2021), 47.

Complete Biconservative Manifolds

Aykut Kayhan

Mathematics and Science Education, Maltepe University, İstanbul, Türkiye,
aykutkayhann@gmail.com

Abstract

Let (M^m, g) and (N^n, h) be two Riemannian manifolds. A critical point of the bienergy functional

$$E_2(\phi) = \frac{1}{2}\int_M |\tau(\phi)|^2 v_g,$$

where $\tau(\phi) = \nabla d\phi(\cdot,\cdot)$ is the tension field of a smooth map $\phi : M \to N$, is called biharmonic map, and it is the characterized by the vanishing of the bitension field $\tau_2(\phi)$. A submanifolds with biharmonic map are called biharmonic submanifolds. In 1924, D. Hilbert called the *stress-energy tensor* associated to a functional E, a symmetric 2-covariant tensor S which is conservative, i.e., $\operatorname{div} S = 0$, at the critical points of E. In the case of the bienergy functional E_2, G. Y. Jiang defined in 1987 the stress-bienergy tensor S_2 and proved that it satisfies $\operatorname{div} S_2 = -\langle \tau_2(\phi), d\phi\rangle$. Thus, if ϕ is biharmonic, then $\operatorname{div} S_2 = 0$ (see [1]). For biharmonic submanifolds, from the above relation, we see that $\operatorname{div} S_2 = 0$ if and only if the tangent part of the bitension field vanishes. A submanifold M is called *biconservative* if $\operatorname{div} S_2 = 0$. In this work, we study complete biconservative hypersurface in the de Sitter space and obtain some rigidity results by using well-known Cheng-Yau formula.

Keywords: biconservative, complete, spacelike.

References

[1] G.Y. Jiang, The conservation law for 2-harmonic maps between Riemannian manifolds, *Acta Math. Sinica* **30** (1987), 220–225.

4x4 Matrix Representation of Hybrid Numbers, Gram Matrix and Golden Ratio

Abdullah Mağden

Department of Mathematics, Bursa Technical University, Bursa, Türkiye,
abdullah.magden@btu.edu.tr

Abstract

This study investigates the Lie algebra structure and geometric properties of hybrid numbers, under a specific non-associative multiplication table. Within the scope of the study, the structure constants of the system were derived through commutator relations, and 4x4 matrix representation was constructed. By defining the Gram matrix, complex and hybrid conjugates, the connection of the eigenvalues of Gram matrices to the golden ratio has been obtained. Theoretical analysis reveals that this hybrid Lie algebra possesses a semi-simple structure and that the characteristic spectrum of its Killing form is directly related to the golden ratio φ. Furthermore, it is proven that the complex and hyperbolic conjugation operations are symmetry transformations that leave the Killing form invariant.

Keywords: Hybrid numbers, Gram matrix, Killing form, golden ratio.
2010 Mathematics Subject Classification: 11B39, 15A18, 17B81.

References

[1] M. Özdemir, Introduction to hybrid numbers, *Advances in Applied Clifford Algebras* **24**(3) (2014), 827-840.

[2] B. C. Hall, Lie Groups, Lie Algebras and Representations, Springers International Publishing, 2015.

[3] R. Gilmore, Lie Groups, Physics and Geometry, An Introduction for Physicsts, Engineers and Chemists. Cambridge University Press, 2008.

[4] S. Falcon, A. Plaza, The metallic ratios family and Golden ratio, *Chaos, Solitons & Fractals* **33**(3) (2007), 757-765.

[5] İ. Öztürk, M. Özdemir, Similarity of hybrid numbers, Mathematical Methods in the Applied Sciences **43**(15) (2020), 8867-8881.

[6] J. E. Humphrey, Introduction to Lie Algebras and Representation Theory, Springer-Verlag. New York Inc, 1972.

[7] D. B. Sweetser, Doing physics with quternions, MIT, US. (2005).

[8] A. L. Malagon, Killing Form of Lie Algebras, Emory University ProQuest Dissetations& Theses (2000).

[9] R. A. Horn, C. R. Johnson, Matrix Analysis. Cambridge University Press, 2013.

Geometric Applications in Dual Transformations

Gülsüm Yüca, Yusuf Yaylı

Aksaray University, Aksaray, Türkiye, gulsumbicer@gmail.com
Ankara University, Ankara, Türkiye, yayli@science.ankara.edu.tr

Abstract

This research examines the relationship between Euclidean and Lorentzian spaces via dual transformation theory, systematically linking the rotation groups $SO(n)$ and $SO(n-1,1)$ [1, 4]. This study provides a unified synthesis of dual transformations. Initially, semi-orthogonal matrix representations of motion in 3D and 4D spaces are derived. Subsequently, real and dual quaternions are incorporated into the analysis to characterize these transformations [3, 5]. Finally, this methodology is extended to hyper-dual matrices, demonstrating that current dual transformations can be generalized into a more inclusive structure [2]. Additionally, this modeling is visualized through comparative projections of surfaces across Euclidean and Lorentzian spaces [2, 6]. Dual transformation serves as a bridge between these spaces, enabling the systematic investigation of rigid body motions[1, 4]. This research presents a new analytical approach to motion and offers theoretical insights into the geometric transitions across these spaces [3, 5, 6].

Keywords: Dual transformation, kinematics, rotation matrices, semi-orthogonal matrices.
2020 Mathematics Subject Classification: 53A17, 70B10.

References

[1] R. Dohi, Y. Maeda, M. Mori, H. Yoshida, A dual transformation between $SO(n+1)$ (00-component $= 0$) and $SO(n,1)$ and its geometric applications, *Linear Algebra Appl.* **432** (2010), 770–776.

[2] G. Yüca, Y. Yaylı, Hyper-dual matrices and dual transformations, *J. Geom. Phys.* **175** (2022), 104473.

[3] G. Yüca, Y. Yaylı, Dual transformations and quaternions, *Math. Methods Appl. Sci.* **44** (2021), 10957–10971.

[4] G. Yüca, Y. Yaylı, Dual transformation between $S\hat{O}(3)$ and $S\hat{O}(2,1)$ and its geometric applications, *Proc. Natl. Acad. Sci., India, Sect. A Phys. Sci.* **88** (2018), 267–273.

[5] G. Yüca, Kinematics applications of dual transformations, *J. Geom. Phys.* **163** (2021), 104139.

[6] G. Yüca, Y. Yaylı, Quaternions and homothetic motions in Euclidean and Lorentzian spaces, *Honam Math. J.* **45** (2023), 198–214.

Parameter-Free Filled Functions in Global Optimization: Theory and Benchmark Testing

Gülden Kapusuz

Department of Basic Sciences, Isparta University of Applied Sciences, Isparta, Türkiye
guldenkapusuz92@gmail.com

Abstract

This paper introduces a new continuously differentiable, parameter-free filled function for unconstrained global optimization. While traditional filled functions are effective in escaping local minimizers, they frequently rely on sensitive, problem-dependent parameters and often exhibit non-smoothness. These drawbacks limit the applicability of fast, gradient-based local minimization algorithms. The proposed function circumvents these structural issues. We analytically prove its core properties, demonstrating that it systematically modifies the objective landscape to move past current local minima without requiring any parameter tuning. To validate the theoretical framework, the proposed method is tested on standard benchmark optimization problems. Numerical results confirm that the smoothness and parameter-free nature of the proposed approach yield a highly stable and efficient global search mechanism.

Keywords: Filled function, parameter-free, global optimization, continuously differentiable.
2010 Mathematics Subject Classification: 65K05, 90C26, 90C30.

References

[1] A. Şahiner, N. Yılmaz, G. Kapusuz, A novel modeling and smoothing technique in global optimization, *Journal of Industrial and Management Optimization* **15** (2019), 113–130.

[2] R. P. Ge, The theory of filled function method for finding global minimizers of nonlinearly constrained minimization problems, *Journal of Computational Mathematics* **5** (1987), 1–9.

[3] R. P. Ge, A filled function method for finding a global minimizer of a function of several variables, *Mathematical Programming* **46** (1990), 191–204.

[4] Y. Shang, D. Qu, J. Li, R. Zhang, A new parameter-free continuously differentiable filled function algorithm for solving nonlinear equations and data fitting problems, *Journal of Computational and Applied Mathematics* **454** (2025), 116198.

[5] R. Pandiya, A. Ahdika, A. Sahiner, B. Pratikno, EDECT algorithm: The hybridization of the filled function and DIRECT methods for global optimization, *Numerical Algorithms* (2026), 1–34.

[6] X. Liu, Finding global minima with a computable filled function, *Journal of Global Optimization* **19** (2001), 151–161.

Graph-Type General Helices on a Right Circular Cylinder: Characterization, Geodesicity, and Planarity

Ahmet Mollaoğulları

Department of Mathematics, Faculty of Science, Çanakkale Onsekiz Mart University, Çanakkale, Türkiye, ahmet_m@comu.edu.tr

Abstract

We study curves of the form $\gamma(\theta) = (R\cos\theta,\ R\sin\theta,\ f(\theta))$ lying on the right circular cylinder $x^2 + y^2 = R^2$, where $R > 0$ and f is of class C^3. By computing the curvature and torsion of this curve family and applying Lancret's characterization, we obtain an explicit necessary and sufficient condition for such a curve to be a general helix. We also characterize the geodesic and planar members of the same class via differential conditions on f. Finally, we show that the general helix condition implies neither geodesicity nor planarity by constructing an explicit example of a non-geodesic and non-planar general helix.

Keywords: General helix, right circular cylinder, geodesic curve, planar curve.
2010 Mathematics Subject Classification: 53A04, 53A05.

References

[1] M. Barros, General helices and a theorem of Lancret, *Proc. Amer. Math. Soc.* **125** (1997), 1503–1509.

[2] M. P. do Carmo, Differential Geometry of Curves and Surfaces, Prentice-Hall, Englewood Cliffs, 1976.

[3] A. Pressley, Elementary Differential Geometry, Springer, London, 2010.

Generalized Magnetic Curves in the 3-Dimensional Heisenberg Group

Cihan Özgür

Department of Mathematics, İzmir Demokrasi University, İzmir, Türkiye,
cihan.ozgur@idu.edu.tr

Abstract

In the present study, we consider generalized magnetic curves in the 3-dimensional Heisenberg group endowed with its natural contact metric structure. We characterize non-geodesic generalized magnetic curves by reducing the generalized Lorentz force equation to a system of ordinary differential equations. We obtain explicit expressions for the curvature and torsion of non-geodesic generalized magnetic curves and derive local parametrizations for these curves.

Keywords: Generalized magnetic curve, Magnetic curve, Heisenberg group, Contact metric manifold.
2010 Mathematics Subject Classification: 53C15, 53C25, 53C50.

References

[1] P. Alegre, J. Barrera, A. Carriazo and D. García-Cano, A generalization of magnetic curves in contact metric geometry, *Results in Math.* **79**(4) (2024), 141, 18.

[2] J. L. Cabrerizo, M. Fernández, J. S. Gómez, On the existence of almost contact structure and the contact magnetic field, *Acta Math. Hungar.* **125**(1-2) (2009), 191–199.

[3] S. L. Druţă-Romaniuc, J. Inoguchi, M. I. Munteanu, A. I. Nistor, Magnetic curves in Sasakian manifolds. *J. Nonlinear Math. Phys.* **22**(3) (2015), 428–447.

[4] C. Özgür, On magnetic curves in the 3-dimensional Heisenberg group, *Proc. Inst. Math. Mech. Natl. Acad. Sci. Azerb.* **43**(2) (2017), 278–286.

On Anti-Invariant Submanifolds of LP-Sasakian Manifolds

Hülya Aytimur, Cihat Kılınç

Department of Mathematics, Balıkesir University, Balıkesir, Türkiye,
hulya.aytimur@balikesir.edu.tr
Department of Mathematics, Balıkesir University, Balıkesir, Türkiye,
cihatkilincoo8@hotmail.com

Abstract

In this study, the fundamental properties of LP-Sasakian manifolds are examined, and the conditions under which anti-invariant submanifolds of such manifolds are totally geodesic, minimal, and parallel are investigated. The approach adopted in [4] is followed in deriving the results.

Keywords: Submanifolds, anti-invariant, LP-Sasakian.
2010 Mathematics Subject Classification: 53C15, 53C25, 53C40.

References

[1] T. Adati, T. Miyazawa, On P-Sasakian manifolds satisfying certain conditions, *Tensor (N. S.)* **33** (1979), 173–178.

[2] U. C. De, D. Tarafdar, On a type of P-Sasakian manifold, *Balkanica* **7** (1993), 211–215.

[3] I. Mihai, R. Rosca, On Lorentzian P-Sasakian manifolds, in Classical Analysis World Scientific, 1992, pp. 155–169,

[4] S. Dirik, M. Atçeken, Ü. Yıldırım, Anti-invariant submanifolds of a normal paracontact metric manifold, *GJOM* **10**(2) (2021), 38-49.

Bertrand Offsets of Ruled Surfaces in 3-Dimensional Riemannian Manifolds

Kemalettin Can DENİZ, Ahmet YÜCESAN, Gözde ÖZKAN TÜKEL, Tunahan TURHAN

Graduate School of Natural and Applied Sciences, Süleyman Demirel University, Isparta, TÜRKİYE, yl2530110009@ogr.sdu.edu.tr

Faculty of Engineering and Natural Sciences, Süleyman Demirel University, Isparta, TÜRKİYE, ahmetyucesan@sdu.edu.tr

Faculty of Technology, Isparta University of Applied Sciences, Isparta, TÜRKİYE, gozdetukel@isparta.edu.tr

Faculty of Education, Süleyman Demirel University, Isparta, TÜRKİYE, tunahanturhan@sdu.edu.tr

Abstract

In this study, Bertrand offsets of ruled surfaces in 3-dimensional Riemannian manifolds are investigated. For this purpose, first, the Bertrand offsets of ruled surfaces are defined along the striction curve of the surface by using the Sannia frame and the exponential map. Then, for the Bertrand offsets of ruled surfaces in simply connected space forms, the angular offset and the distance offset are shown to be constant. Thus, it is obtained that Bertrand offset pairs in simply connected space forms have a geometric relation with constant angle and constant distance. Finally, the condition ensuring the developability of the offset surfaces is obtained in terms of the solutions of the orthogonal Jacobi equation and the invariants of the Sannia frame defined along the surfaces.

Keywords: Bertrand offset, developable surface, ruled surface, simply connected space forms, striction curve.

2020 Mathematics Subject Classification: 53A05, 53B20.

(*) This work is supported by the TÜBİTAK project numbered 125F489.

References

[1] J. Bertrand, *La théorie des courbes à double courbure*, Journal de Mathématiques Pures et Appliquées **15** (1850), 332–350.

[2] J. H. Choi, T. H. Kang, and Y. H. Kim, *Bertrand curves in 3-dimensional space forms*, Appl. Math. Comput. **219** (2012), no. 3, 1040–1046.

[3] L. C. B. da Silva and J. D. da Silva, *Characterization of manifolds of constant curvature by ruled surfaces*, São Paulo J. Math. Sci. **16** (2022), no. 2, 1138–1162.

[4] L. C. da Silva, G. S. Ferreira Jr., and J. D. da Silva, *Curves and surfaces making a constant angle with a parallel transported direction in Riemannian spaces*, arXiv preprint arXiv:2403.10716 (2024).

[5] M. C. López, M. E. Rosado, and A. Soria, *Ruled surfaces in* 3*-dimensional Riemannian manifolds*, Mediterr. J. Math. **21** (2024), no. 3.

[6] B. Ravani and T. S. Ku, *Bertrand offsets of ruled and developable surfaces*, Comput.-Aided Des. **23** (1991), no. 2, 145–152.

[7] M. Spivak, *A Comprehensive Introduction to Differential Geometry, Vol. III*, 3rd ed., Publish or Perish, 1979.

[8] D. Struik, *Lectures on Classical Differential Geometry*, Dover Publications, Inc., New York, 1988.

[9] K. Yano and M. Kon, *Structures on Manifolds*, World Scientific, Singapore, 1984.

Some Results on Kähler–Norden Statistical Manifolds

Gökhan Arıkan, H. Bayram Karadağ

Institute of Science,İnönü University, Malatya, Türkiye, g.arikan1@hotmail.com
Faculty of Arts and Sciences, Department of Mathematics, İnönü University, Malatya, Türkiye, bayram.karadag@inonu.edu.tr

Abstract

Statistical manifolds arise naturally in information geometry through the interaction of a pseudo-Riemannian metric and a pair of dual affine connections. On the other hand, Kähler–Norden manifolds constitute an important class of almost complex manifolds equipped with a Norden metric. In this work, we investigate several geometric properties of Kähler–Norden statistical manifolds. In particular, the relationships among the statistical connection, its dual connection, the almost complex structure, and the associated twin metric are examined. Furthermore, invariance properties of the statistical curvature tensor under the action of the complex structure are established. We also discuss conditions under which the twin metric induces a Kähler–Norden statistical structure and provide illustrative examples. These results contribute to a better understanding of the interplay between Norden geometry and statistical geometry and may serve as a basis for further studies in information geometry and complex differential geometry.

Keywords: Statistical manifolds, Norden structure, Kaehler structure, dual connection.
2010 Mathematics Subject Classification: 53B20, 53C25.

References

[1] S. Amari, *Differential-Geometrical Methods in Statistics*, Lecture Notes in Statistics, Vol. 28, Springer-Verlag, Berlin, 1985.

[2] H. Furuhata, I. Hasegawa, Y. Okuyama, K. Sato, M. H. Shahid, Sasakian Statistical Manifolds, *Journal of Geometry and Physics*, **117** (2017), 179–186.

[3] E. Peyghan, A. Gezer and L. Nourmohammadifar, Kähler-Norden Structures on Statistical Manifolds, *Filomat*, **36**(17) (2022), 5691–5706.

[4] H. Aytimur, I. Mihai, C. Özgür, Relations between Extrinsic and Intrinsic Invariants of Statistical Submanifolds in Sasaki-Like Statistical Manifolds, *Mathematics* **9**(11) (2021), 1285.

A Study on Quaternionic Framed Curves

Ayşe Zeynep Azak, Tülay Erişir, Arzu Sürekçi, Mehmet Ali Güngör

Department of Mathematics and Science Education, Sakarya University, Sakarya, Türkiye, apirdal@sakarya.edu.tr
Department of Mathematics, Erzincan Binali Yıldırım University, Erzincan, Türkiye, tulay.erisir@erzincan.edu.tr
Department of Vocational School of Computer Technologies, Sakarya University of Applied Sciences, Sakarya, Türkiye, arzusurekci@subu.edu.tr
Department of Mathematics, Sakarya University, Sakarya, Türkiye, agungor@sakarya.edu.tr

Abstract

We have introduced a new approach for quaternionic framed curves with singular points. We have expressed the curvatures of these quaternion curves using this new approach. Finally, we have provided an example of this approach.

Keywords: Framed curve, quaternionic curve, singular points.
2010 Mathematics Subject Classification: 53A04, 53A40.

References

[1] M. Akyiğit, Ö. G. Yıldız, On the framed normal curves in Euclidean 4-space, *Fund. J. Math. Appl.* **4**(4) (2021), 258–263.

[2] K. Bharathi, M. Nagaraj, Quaternion valued function of a real variable Serret-Frenet formula, *Indian J. Pure Appl. Math.* **18** (1987), 507–511.

[3] Z. Derin Yaqub, M. A. Güngör, Serret-Frenet formula of quaternionic framed curves, *Math. Sci. Appl. E-Notes* **13** (2025), 92–105.

[4] B. Doğan Yazıcı, S. Özkaldı Karakuş, M. Tosun, Characterizations of framed curves in four-dimensional Euclidean space, *Univ. J. Math. Appl.* **4** (2021), 125–131.

[5] B. Doğan Yazıcı, S. Özkaldı Karakuş, M. Tosun, On the classification of framed rectifying curves in Euclidean space, *Math. Methods Appl. Sci.* **45** (2021), 12089–12098.

[6] B. Doğan Yazıcı, S. Özkaldı Karakuş, M. Tosun, Framed curves and their applications based on a new differential equation, *Int. Electron. J. Geom.* **15** (2022), 47–56.

[7] T. Fukunaga, M. Takahashi, Framed surfaces and one-parameter families of framed curves in Euclidean 3-space, *J. Singul.* **21** (2020), 30–49.

[8] S. Honda, M. Takahashi, Framed curves in the Euclidean space, *Adv. Geom.* **16** (2016), 265–276.

[9] S. Honda, M. Takahashi, Bertrand and Mannheim curves of framed curves in the 3-dimensional Euclidean space, *Turkish J. Math.* **44** (2020), 883–899.

[10] F. Kahraman Aksoyak, A new type of quaternionic frame in R^4, *Int. J. Geom. Methods Mod. Phys.* **16** (2019), 1950084.

[11] Y. Li, K. Eren, K. H. Ayvacı, S. Ersoy, The developable surfaces with pointwise 1-type Gauss map of Frenet-type framed base curves in Euclidean 3-space, *AIMS Math.* **8** (2023), 2226–2239.

[12] Y. Wang, D. Pei, R. Gao, Generic properties of framed rectifying curves, *Mathematics* **7(1)** (2019), 37.

Some New Characterization Results on Lightlike Submanifolds in Indefinite Contact Geometry

Burçin Doğan

Department of Engineering Basic Sciences, Faculty of Engineering and Natural Sciences, Malatya Turgut Özal University, Malatya, Türkiye,
burcin.dogan@ozal.edu.tr

Abstract

In this work, we study new classes of lightlike submanifolds of indefinite Kenmotsu manifolds. The relations between the contact structure, degenerate distributions and transversal bundles are investigated. Several characterization results are obtained in terms of the second fundamental form and shape operators. The results provide new examples and properties of lightlike geometry in the indefinite contact setting.

Keywords: Lightlike submanifold, indefinite contact geometry, screen distribution, second fundamental form, shape operator.
Mathematics Subject Classification 2020: 53C15, 53C25, 53C40, 53C42.

References

[1] K. L. Duggal, A. Bejancu, *Lightlike Submanifolds of Semi-Riemannian Manifolds and Applications*, Kluwer Academic Publishers, Dordrecht, 1996.

[2] K. L. Duggal, B. Şahin, *Differential Geometry of Lightlike Submanifolds*, Birkhäuser Verlag, Basel, 2010.

[3] K. Kenmotsu, A class of almost contact Riemannian manifolds, *Tohoku Mathematical Journal* **24** (1972), 93-103.

[4] B. Doğan, B. Şahin, E. Yaşar, Screen transversal Cauchy–Riemann lightlike submanifolds, *Filomat*, **34**(5) (2020), 1581–1599.

[5] B. Şahin, Screen transversal lightlike submanifolds of indefinite Kähler manifolds, *Chaos, Solitons & Fractals* **38** (2008), 1439–1448.

[6] K. L. Duggal, B. Şahin, Generic lightlike submanifolds of indefinite Sasakian manifolds, *International Journal of Mathematics and Mathematical Sciences*, 2007, Article ID 87063.

Slant Ruled Surfaces in 3-Dimensional Riemannian Manifolds

Hatice Sena Savcu, Tunahan Turhan, Ahmet Yücesan, Gözde Özkan Tükel

Graduate School of Natural and Applied Sciences, Süleyman Demirel University, Isparta, Türkiye, yl2430110010@ogr.sdu.edu.tr
Faculty of Education, Süleyman Demirel University, Isparta, Türkiye, tunahanturhan@sdu.edu.tr
Faculty of Engineering and Natural Sciences, Süleyman Demirel University, Isparta, Türkiye, ahmetyucesan@sdu.edu.tr
Faculty of Technology, Isparta University of Applied Sciences, Isparta, Türkiye, gozdetukel@isparta.edu.tr

Abstract

In this study, slant ruled surfaces in 3-dimensional Riemannian manifolds are defined and their geometric structure is investigated. Using the Sannia frame, X_1, X_2 and X_3 slant ruled surfaces are defined through the condition that one of the frame vectors makes a constant angle with a parallel transported unit vector field. These surfaces are characterized in terms of the Sannia invariants. It is shown that X_1 and X_3 slant surfaces are equivalent to the constancy of the ratio of the Sannia invariants, whereas X_2-slant surfaces are equivalent to the constancy of a certain invariant function. The main result of the study is that X_2-slant ruled surfaces are precisely the rectifying surfaces whose base curve is a slant helix. It is further shown that these surfaces are developable and that the Sannia invariants correspond bijectively to the Frenet curvatures of the base curve. Finally, it is shown that no nontrivial X_2-slant ruled surface exists in simply connected space forms of nonzero constant sectional curvature.

Keywords Ruled surface, simply connected space form, Sannia frame, developable ruled surface, parallel transported unit vector field.
2020 Mathematics Subject Classification: 53B20, 53A25, 53A05.
Acknowledgements: This work is supported by the TUBITAK project numbered 125F489

References

[1] M. Önder, Slant ruled surfaces, *arXiv preprint arXiv:1311.0627* (2013).

[2] M. Spivak, *A Comprehensive Introduction to Differential Geometry, Vol. III*, 3rd ed., Publish or Perish, 1979.

[3] L. C. B. da Silva, G. S. Ferreira Jr, J. D. da Silva, Curves and surfaces making a constant angle with a parallel transported direction in Riemannian spaces, *arXiv preprint arXiv:2403.10716* (2024).

[4] H. A. Hayden, On a generalized helix in a Riemannian n-space, *Proceedings of the London Mathematical Society*, **2**(1) (1931), 337-345.

[5] L. C. B. da Silva, J. D. da Silva, Characterization of manifolds of constant curvature by ruled surfaces, *São Paulo J. Math. Sci.* **16**(2) (2022), 1138-1162.

[6] M. Castrillón López, M. E. Rosado, A. Soria, Ruled surfaces in 3-dimensional Riemannian manifolds, *Mediterranean Journal of Mathematics*, **21**(3) (2024), 97.

[7] S. Izumiya, N. Takeuchi, New special curves and developable surfaces, *Turkish Journal of Mathematics*, **28**(2) (2004), 153-164.

[8] D. J. Struik, *Lectures on Classical Differential Geometry*, Dover Publications, New York, 1988.

[9] M. P. do Carmo, *Differential Geometry of Curves and Surfaces: Revised and Updated Second Edition*, Courier Dover Publications, 2016.

[10] Y. C. Wong, On the generalized helices of Hayden and Syptak in an N-space, *Proc. Cambridge Philos. Soc.* **37** (1941), 229-243.

[11] K. Yano, M. Kon, *Structures on Manifolds*, World Scientific, Singapore, 1984.

Mannheim Offsets of Ruled Surfaces in Simply Connected 3-Dimensional Space Forms

Yağmur Karabıyık, Gözde Özkan Tükel, Tunahan Turhan, Ahmet Yücesan

Graduate School of Natural and Applied Sciences, Süleyman Demirel University, Isparta, Türkiye, yl2430110008@ogr.sdu.edu.tr

Faculty of Technology, Isparta University of Applied Sciences, Isparta, Türkiye, gozdetukel@isparta.edu.tr

Faculty of Education, Süleyman Demirel University, Isparta, Türkiye, tunahanturhan@sdu.edu.tr

Faculty of Engineering and Natural Sciences, Süleyman Demirel University, Isparta, Türkiye, ahmetyucesan@sdu.edu.tr

Abstract

We define the Mannheim offsets of ruled surfaces in 3-dimensional Riemann manifolds and investigate their fundamental properties in simply connected space forms. We construct these offsets by means of the exponential map, parallel transport, and the Sannia frame. We give necessary and sufficient conditions for the constancy of the distance offset and the angular offset. We further characterize the developability of the Mannheim offset in terms of the Sannia invariants.

Keywords: Mannheim offset, ruled surface, Riemannian manifold, Sannia frame, developable surface.
2020 Mathematics Subject Classification: 53A05, 53B20.
Acknowledgements: This work is supported by the TUBİTAK project numbered 125F489.

References

[1] J. H. Choi, T. H. Kang, Y. H. Kim, Mannheim curves in 3-dimensional space forms, *Bull. Korean Math. Soc.* **50**(4) (2013), 1099–1108.

[2] L. C. B. da Silva, J. D. da Silva, *Characterization of manifolds of constant curvature by ruled surfaces*, *São Paulo J. Math. Sci.* **16**(2) (2022), 1138–1162.

[3] H. Liu, F. Wang, Mannheim partner curves in 3-space, *Journal of Geometry* **88**(1–2) (2008), 120–126.

[4] M. C. López, M. E. Rosado, A. Soria, Ruled surfaces in 3-dimensional Riemannian manifolds, *Mediterr. J. Math.* **21**(3) (2024).

[5] K. Orbay, E. Kasap, İ. Aydemir, Mannheim offsets of ruled surfaces, *Math. Probl. Eng.* **2009** (2009), Article ID 160917.

[6] M. Spivak, A Comprehensive Introduction to Differential Geometry, Vol. III, 3rd ed., Publish or Perish, 1979.

[7] K. Yano, M. Kon, Structures on Manifolds, World Scientific, Singapore, 1984.

[8] F. Wang, H. Liu, Mannheim partner curves in 3-Euclidean space, *Mathematics in Practice and Theory* **37**(1) (2007), 141–143.

Mannheim Offsets of Ruled Surfaces with B-Darboux Frame

Faik Duran, Gül Uğur Kaymanlı

Department of Mathematics, Çankırı Karatekin University, Çankırı, Turkey, duranfaik18@gmail.com

Department of Mathematics, Çankırı Karatekin University, Çankırı, Turkey, gulugurk@karatekin.edu.tr

Abstract

This paper introduces the Mannheim offset of a ruled surface associated with the B-Darboux frame. By using the B-Darboux frame, fundamental geometric properties of the Mannheim offset surface, such as developability, the striction curve, and the distribution parameter, are investigated. In addition, the Gaussian and mean curvatures are obtained. Several examples are included to illustrate the theoretical results, and the corresponding surfaces are visualized.

Keywords: Curvatures, offsets, ruled surface, B-Darboux frame.
2010 Mathematics Subject Classification: 53A05, 57R25.

References

[1] M. Dede, C. Ekici, A. Görgülü, Directional q-frame along a space curve, *Int. J. Adv. Res. Comput. Sci. Softw. Eng.* **5**(12) (2015), 775–780.

[2] M. Kazaz, H. H. Uğurlu, M. Önder, T. Kahraman, Mannheim partner D-curves in the Euclidean 3-space, *New Trends Math. Sci.* **3** (2015), 24–35.

[3] G. Uğur Kaymanlı, Characterization of the evolute offset of ruled surfaces with B-Darboux frame, *J. New Theory* **33** (2020), 50–55.

[4] K. Orbay, E. Kasap, İ. Aydemir, Mannheim offsets of ruled surfaces, *Math. Probl. Eng.* (2009), Article ID 160917, 9.

[5] M. Önder, H. H. Uğurlu, Some results and characterizations for Mannheim offsets of the ruled surfaces, *Bol. Soc. Parana. Mat.* **34**(1) (2015), 85–98.

[6] G. Y. Saçlı, S. Yüce, A method for Mannheim offsets of ruled surfaces, *J. Sci. Arts* **18**(4) (2018), 827–836.

[7] G. Y. Şentürk, S. Yüce, Characteristic properties of the ruled surface with Darboux frame in E^3, *Kuwait J. Sci.* **42**(2) (2015), 14–33.

Characterizations of Bi-Rotational Hypersurface in Euclidean 4-Space

Neriman Acar, Ferdağ Kahraman Aksoyak

Department of Mathematics, Ahi Evran University, Kırşehir, Türkiye, nerimanacar_07@hotmail.com

Department of Elementary Mathematics Education, Ahi Evran University, Kırşehir, Türkiye, ferdag.aksoyak@ahievran.edu.tr

Abstract

In this paper, we study bi-rotational hypersurfaces and obtain some characterizations based on the fact that such surfaces are flat or minimal. Also, we give some examples of these surfaces by using Mathematica.

Keywords: Euclidean space, rotational surface, mean and Gaussian curvature.
2010 Mathematics Subject Classification: 53B25, 53C40.

References

[1] C. Moore, Surfaces of rotation in a space of four dimensions, *Ann. Math.* **21** (1919), 81–93.

[2] C. Moore, Rotation surfaces of constant curvature in space of four dimensions, *Bull. Amer. Math. Soc.* **26** (1920), 454–460.

[3] E. Güler, Rotational hypersurfaces satisfying $\Delta^I R = AR$ in the four-dimensional Euclidean space, *Politeknik Dergisi* **24** (2021), 517–520.

[4] E. Güler, Y. Yaylı, H. H. Hacısalihoğlu, Bi-rotational hypersurface with $\Delta x = Ax$ in 4-space, *Facta Univ. Ser. Math. Inform.* **37** (2022), 917–928.

[5] E. Güler, Y. Yaylı, H. H. Hacısalihoğlu, Bi–rotational hypersurfaces satisfying $\Delta^{III} x = Ax$ in 4-space, *Honam Mathematical J.* **44** (2022), 219–230.

Pointwise Quasi Hemi-Slant Riemannian Maps into Almost Hermitian Manifolds and Casorati Inequalities

Sezin Aykurt Sepet, Meltem Karaismailoğlu, Mahmut Ergüt

Department of Mathematics, Kırşehir Ahi Evran University, Kırşehir, Türkiye, saykurt@ahievran.edu.tr
Department of Mathematics, Kırşehir Ahi Evran University, Kırşehir, Türkiye, mltm.33.27@hotmail.com
Department of Mathematics, Tekirdağ Namık Kemal University, Tekirdağ, Türkiye, mergut@nku.edu.tr

Abstract

In the present paper, we introduce a new class of Riemannian maps, called pointwise quasi hemi-slant (PQHS) Riemannian maps, from Riemannian manifolds into almost Hermitian manifolds. This class provides a natural generalization of invariant, anti-invariant, and pointwise slant structures. We investigate the geometric structure of the foliations determined by the distributions associated with PQHS Riemannian maps. Moreover, we establish necessary and sufficient conditions for such maps to be totally geodesic. Finally, we obtain Casorati curvature inequalities for PQHS Riemannian maps to complex space forms. These results contribute to the existing literature by extending the theory of Casorati type inequalities in the setting of Riemannian maps.

Keywords: Riemannian map, almost Hermitian manifold, pointwise quasi hemi-slant Riemannian map, Casorati inequlity.
2010 Mathematics Subject Classification: 53C40, 53C43.

References

[1] M. A. Akyol, Y. Gündüzalp, Pointwise hemi-slant Riemannian maps (PHSRM) from almost Hermitian manifolds, *Hacet. J. Math. Stat.* **53**(5) (2024), 1218–1237.

[2] F. Casorati, Nuova definizione della curvatura delle superficie e suo confronto con quella di Gauss (New definition of the curvature of the surface and its comparison with that of Gauss), *Rend. Inst. Matem. Accad. Lomb. Ser. II* **22**(8) (1889), 335–346.

[3] Y. Gündüzalp, M. A. Akyol, B. Şahin, Pointwise hemi-slant Riemannian maps into almost Hermitian manifolds and Casorati inequalities, *International Journal of Maps in Mathematics* **7**(1) (2024), 76–96.

[4] C. W. Lee, J. W. Lee, B. Şahin, G. E. Vilcu, Optimal inequalities for Riemannian maps and Riemannian submersions involving Casorati curvatures, *Ann. Mat. Pura Appl.* **200** (2021), 1277–1295.

A Homothetic Hodge–de Rham Theory and Geometric Regularization of Elliptic Boundary Value Problems

Fereidoun Sabetghadam
Mechanical Eng. Dept., Science and Research Branch, IAU, Tehran, Iran,
fsabet@srbiau.ac.ir

Abstract

Building on our recent work, we introduce a homothetic extension of classical Weyl–integrable geometry. The construction generalizes the usual linear gauge transformations to affine homothetic transformations centered at a distinguished harmonic, scale-invariant form α_d. After a suitable shift of variables, these affine transformations become linear, leading to a twisted exterior calculus that is structurally equivalent to the Witten deformation of the de Rham complex. In this setting we develop a homothetic Hodge theory by defining a twisted adjoint and a homothetic Laplacian, and we prove a corresponding Hodge decomposition theorem on compact Riemannian manifolds. As an application to partial differential equations, we show that the scalar homothetic Laplacian yields a rigorous diffuse-interface (volume-penalization) formulation of elliptic boundary value problems. When the Weyl scale field is modeled by a fixed distribution concentrated near a hypersurface, the resulting lower-order geometric terms generate a penalty layer that enforces Dirichlet, Neumann, or Cauchy data within a single geometric equation. This framework also yields consistent weak solutions in situations involving classically incompatible Cauchy data. Finally, we apply the theory to construct a nonsingular model for point sources in elliptic field equations, preserving the correct Coulomb far field while removing the core singularity and producing finite field energy.

Keywords: Weyl-integrable geometry, Witten deformation, weighted Hodge theory, twisted de Rham complex, homothetic Laplacian, elliptic boundary value problems.
2020 Mathematics Subject Classification: 35J25, 53C07, 58A12, 58J05.

References

[1] F. Sabetghadam, Homothetic Hodge−de Rham theory and a geometric regularization of elliptic boundary value problems, *Journal of Geometry and Physics* (under review), arXiv:2603.27564, (2026).

[2] F. Sabetghadam, A homothetic gauge theory and the regularization of the point charge, *International Journal of Geometric Methods in Modern Physics* (under review), arXiv:2507.06153, (2025).

The Algebraic Structure of $H_{(a_1,a_2,a_3)}(\varepsilon)$-Elliptic Quaternions over Z_p

Hatice Çetinkaya, Gülay Koru Yücekaya

Department of Mathematics, Selcuk University Faculty of Science, Department of Mathematics, Konya, Türkiye, haticeeccetinkaya@gmail.com
Department of Mathematics and Science Education, Gazi University, Ankara, Türkiye, gkoru@gazi.edu.tr

Abstract

Quaternions are non-commutative algebraic structures that extend the complex number system. Owing to their capacity to represent rotations in three-dimensional space efficiently, they have found extensive applications in geometry, mechanics, physics, computer graphics, and robotics. In this study, the algebraic structure of $H_{(a_1,a_2,a_3)}(\varepsilon)$-elliptic quaternions over the finite field Z_p, where p is a prime number, is investigated. To this end, an elliptic quaternion multiplication is introduced on the vector space associated with these quaternions. It is shown that the resulting multiplication endows the underlying vector space with the structure of an elliptic quaternion algebra over p. The fundamental algebraic properties of this structure are established. Furthermore, a matrix representation of the obtained algebra is constructed by means of linear transformations, and some algebraic properties of the matrices arising from this representation are investigated.

Keywords: Ring, field, elliptic quaternion, elliptic quaternion algebra.
2010 Mathematics Subject Classification: 15A33, 16K20.

References

[1] H. H. Hacısalihoğlu, Motion Geometry and Quaternions Theory, Gazi University Publications, Ankara, 1983.

[2] M. Özdemir, Quaternions and Geometry, Altın Nokta Publishing, 2020.

[3] R. Ghadami, Y. Yaylı, Elliptic quaternion and elliptic linear interpolation, *Journal of Polytechnic* **27**(3) (2024), 1189–1195.

[4] G. K. Yücekaya, F. Orman, Algebra of elliptical quaternions, *Advanced Studies: Euro-Tbilisi Mathematical Journal* **16**(4) (2023), 9–20.

[5] O. P. Agrawal, Hamilton operators and dual-number quaternions in spatial kinematics, *Mechanism and Machine Theory* **22**(6) (1987), 569–575.

[6] H. B. Çolakoğlu, M. Özdemir, Generalized elliptical quaternions with some applications, *Turkish Journal of Mathematics* **47**(1) (2023), 351-–371.

[7] H. Ceyhan, Z. Özdemir, Multiplicative elliptic quaternions and generating geometric elliptic rotation matrices, *International Journal of Geometric Methods in Modern Physics* **22**(5) (2025).

New Extensions on Involute–Evolute Curves in Euclidean Space and Lorentzian Space

Vildan Özdemir, Gülay Koru Yücekaya

Department of Mathematics, Selcuk University, Konya, Türkiye,
vildanbacak@gmail.com
Department of Mathematics and Science Education, Gazi University, Ankara, Türkiye,
gkoru@gazi.edu.tr

Abstract

In this study, we investigate the geometry of involute and evolute curves in Euclidean n-space $\mathbb{E}^n$ and Lorentzian 4-space $\mathbb{R}^4_1$, extending classical results beyond the well-known three-dimensional Euclidean setting. We introduce a unified framework based on *spherical parameters* $\sigma_i(s) = \langle \alpha(s), e_i(s) \rangle$ and *support functions* $h_i(s) = \langle \alpha(s), N_i(s) \rangle$ defined with respect to the Frenet frame $\{T, N_1, N_2, N_3\}$. Using this framework, we derive eigenvalue-type differential equations that characterize evolute curves in $\mathbb{E}^n$ via their support functions, and we obtain new geometric characterizations of special curve families including ω-curves. We then extend the theory to Lorentzian 4-space $\mathbb{R}^4_1$, treating all three curve types: timelike, spacelike, and null. We further establish a support function theorem for evolutes in $\mathbb{R}^4_1$ with respect to the Lorentzian inner product $\langle , \rangle_1$, proving that the involute–evolute duality persists in this setting. New eigenvalue characterizations of special curves in Lorentzian 4-space, analogous to the Euclidean case, are also obtained.

Keywords: involute curves, evolute curves, Lorentzian space, spherical parameters, support functions, Frenet frame.
2020 Mathematics Subject Classification: 53A04, 53B30.

References

[1] A. T. Altıntaş, Some special curves and Smarandache curves according to Bishop frame in Minkowski 3-space, *Gazi Univ. J. Sci.* **29**(1) (2016), 89–100.

[2] M. Bilici, M. Çalışkan, On the involutes of the spacelike curve with a timelike binormal in Minkowski 3-space, *Int. Math. Forum* **4**(31) (2009), 1497–1509.

[3] B. Bükcü, M. K. Karacan, On the involute and evolute curves of the spacelike curve with a spacelike binormal in Minkowski 3-space, *Int. J. Contemp. Math. Sci.* **2**(5) (2007), 221–232.

[4] A. C. Çöken, Ü. Özdemir, On the involutes of a curve in Minkowski n-space equipped with a semi-Riemannian metric, *Appl. Math. Comput.* **196** (2008), 428–436.

[5] M. Düldül, B. U. Düldül, N. Kuğuk, N. Önder, Extension of the Darboux frame into Euclidean 4-space and its invariants, *Turkish J. Math.* **41** (2017), 1628–1639.

[6] A. Arfah, On evolutes of null Cartan curves in Minkowski 4-space, *J. Phys.: Conf. Ser.* **1722** (2021), 012039.

[7] S. Müller, A. Schwenk-Schellschmidt, U. Simon, Eigenvalue equations in curve theory Part II: Evolutes and involutes, *Results Math.* **50**(1–2) (2007), 109–124.

[8] M. Hanif, Z. H. Hou, A. Nisar, Null Cartan curve's generalized involute and evolute couple in $\mathbb{E}_1^4$, *Int. J. Geom. Methods Mod. Phys.* (2025).

Kinematic Modeling of Spiral Staircase Climbing Motion

Bülent Karakaş, Şenay Baydaş
Department of Mathematics, Faculty of Science Bartın University, Bartın, Türkiye
bk@bartin.edu.tr
Department of Mathematics, Faculty of Science,Van Yüzüncü Yıl University, Van, Türkiye sbaydas@yyu.edu.tr

Abstract

Walking kinematics and stair climbing motions have been studied by us [1, 2]. In complex architectural settings, spiral staircases present a particular challenge because they combine vertical displacement with continuous angular motion. Unlike straight staircase climbing, motion on a spiral staircase involves simultaneous changes in position, orientation, curvature, and equilibrium, requiring a complex kinematic structure that cannot be fully modeled with linear motion models. The aim of this study is to develop a mathematical model of human movement ascent along a spiral staircase. By representing the staircase geometry as a three-dimensional helical trajectory and examining the relevant kinematic quantities, a combined description of the translational and rotational components of the motion is provided. Rotational staircase climbing motion can be given kinematically as progression along a helix (spiral). A person climbing a spiral staircase: rotates along a circle in the horizontal plane and simultaneously ascends along the axis (performs translational motion). Rotation occurs in a plane perpendicular to the helical axis, and the pole of rotation is the intersection point of the plane and the axis. While the axis of rotation of a person climbing a spiral staircase is the axis of the cylinder in which the staircase is located, the poles of rotation are not on the axis. In the kinematic modeling of the spiral staircase climbing motion, the poles of rotation were taken as the right knee, the left knee, and the center point of the waist, and the modeling was done accordingly.

Keywords: Climbing, kinematics, modeling, staircase.
Mathematics Subject Classification: 70B15, 70G45.

References

[1] Ş. Baydaş, B. Karakaş, Kinematics of 2x3R robotics modelling: Climbing the stairs form, *Hadronic Journal* (2010), 303-320.

[2] Ş. Baydaş, B. Karakaş, Kinematics of 2x3R robotics modelling: walking form, *Advances in Applied Mathematical Analysis* **1**(2) (2006), 1-10.

[3] O. Bottema, B. Roth, Theorical Kinematics, North-Holland Publishing Company, Amsterdam, 557, 1979.

[4] R. J. Schilling, Fundamentals of Robotics, Prentice Hall, New Jersey, 425, 1990.

A Study on Quaternionic and Framed Curves

Arzu Sürekçi

Sakarya University of Applied Sciences, Vocational School of Computer Technologies, 54500, Sakarya, Türkiye, arzusurekci@subu.edu.tr

Abstract

In this study, the geometric structures of quaternionic framed curves and quaternionic curves are investigated, and the fundamental properties of these two approaches are compared. Framed curves, which can be employed in cases where the classical Frenet frame is not well-defined, provide a more general framework for analyzing the geometric behavior of curves. Within this context, the definitions and geometric characteristics of quaternionic framed curves, as well as their relationships with quaternionic curves, are examined. The obtained results indicate that the framed approach offers a flexible and effective tool for the study of quaternionic curves.

Keywords: Quaternionic curves, quaternionic framed curves, differential geometry, quaternions.
2010 Mathematics Subject Classification: 53A04, 57R45, 58K05.

References

[1] B. Doğan Yazıcı, S. Özkaldı Sarakuş, M. Tosun, Framed normal curves in Euclidean space, *Tbilisi Math. J.* (2020), 27–37.

[2] Ç. Camcı, A. Uçum, K. İlarslan, A new approach to Bertrand curves in Euclidean 3-space *J. Geom.* **111** (2020), 1-–15.

[3] F. K. Aksoyak, A new type of quaternionic frame in R^4, *Amer. Int. J. Geom. Methods Mod. Phys.* **16** (2019), 1959984.

[4] F. K. Aksoyak, Quaternionic Bertrand curves according to type-2-quaternionic frame in R^4, *Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat.* **71** (2022), 395–406.

[5] K. Bharathi, M. Nagaraj, Quaternion valued function of a real Serret-Frenet formulae, *Indian J. Pure Appl. Math.* **18** (1987), 507–511.

[6] M. Elzawya, M., S. Mosab, Quaternionic Bertrand curves in the Galilean space, *Filomat* **34** (2020), 59–66.

[7] N. Ekmekci, K. İlarslann, On Bertrand curves and their characterization, *Differ. Geom. Dyn. Syst.* **3** (2001), 17–24.

[8] O. Keçilioğlu, K. İlarslan, Quaternionic Bertrand curves in Euclidean 4-space *Bull. Math. Anal. Appl.* **5** (2013), 27–38.

[9] R. L. Bishop, There is more than one way to frame a curve, *Am. Math. Mon.* **82** (1975), 246–251.

[10] S. Ersoy, S., M. Tosun, Timelike Bertrand curves in semi-Euclidean space, *Int. J. Math. Stat.* **14** (2013), 78–89.

[11] S. Honda, M. Takahashi, Bertrand and Mannheim curves of framed curves in the 3-dimensional Euclidean space, *Turk. J. Math.* **44** (2020), 883–899.

[12] S. Honda, M. Takahashi, H. Yu, Bertrand and Mannheim curves of framed curves in the 4-dimensional Euclidean space, *J. Geom.* **114** (2023), 12.

[13] S. Honda, M. Takahashi, Framed curves in the Euclidean space, *Adv. Geom.* **16** (2016), 265–276.

[14] T. Fukunaga, M. Takahashi, Existence conditions of framed curves for smooth curves, *J. Geom.* **108** (2017), 763–774.

[15] Z. İşbilir, K. E. Özen, M. Tosun, Bertrand partner p-trajectories in the Euclidean 3-space E^3, *Commun. Fac. Sci. Univ. Ankara Ser. A1 Math. Stat.* **72** (2023), 216–228.

[16] Z. D. Yakub, M. A. Güngör, Quaternionic framed curves, *Math. Sci. Appl. E-Notes* **13** (2025), 92–105.

On Tubular Surfaces in Four-Dimensional Galilean Space

Hatice Tozak, Cumali Ekici, Mustafa Dede

Department of Strategy, MEB Denizli Provincial Directorate of National Education, Denizli, Türkiye, hatice.tozak@gmail.com
Mathematics and Computer Science, Eskişehir Osmangazi University, Eskişehir, Türkiye, cekici@ogu.edu.tr
Mathematics and Computer Science, Kilis University, Kilis, Türkiye, mustafadede@kilis.edu.tr

Abstract

This paper investigates tubular surfaces in four-dimensional Galilean space and studies their basic differential geometric properties. By using the Frenet frame associated with a center curve in four-dimensional Galilean space, a precise parametrization of the tubular surface is established, expressed in terms of the curvature functions of the generating curve with a constant radius function. Subsequently, explicit expressions for the fundamental geometric properties are derived, and the curvature conditions of the tubular surface in four-dimensional Galilean 4-space is analyzed in detail.

Keywords: Tubular hypersurface, Galilean 4-space, curvatures.
2010 Mathematics Subject Classification: 53A35, 53B30.

References

[1] A. Elsharkawy, N. Elsharkawy, Quasi-position vector curves in Galilean 4-space, *Frontiers in Physics* **12** (2024), Article 1400730.

[2] B. Yağbasan, C. Ekici, H. Tozak, Directional tube surface in Euclidean 4-space, *Hagia Sophia Journal of Geometry* **5**(2) (2023), 18–30.

[3] M. Dede, Tubular surfaces in Galilean space, *Mathematical Communications* **18** (2013), 209–217.

[4] M. Dede, C. Ekici, H. Tozak, Directional tubular surfaces, *International Journal of Algebra* **9**(12) (2015), 527–535.

[5] M. P. Carmo, Differential Geometry of Curves and Surfaces, Prentice-Hall, New Jersey 1976.

[6] N. Macit, M. Akbıyık, S. Yüce, Some new associated curves of an admissible Frenet curve in 3-dimensional and 4-dimensional Galilean spaces, *Romanian Journal of Mathematics and Computer Science* **7**(2) (2017), 110–122.

[7] S. Mosa, M. El-Fakharany, M. Elzawy, Normal curves in 4-dimensional Galilean space G_4, *Frontiers in Physics* **9** (2021), Article 660241.

On Almost Pseudo $\mathcal{B}$-Symmetric Manifolds

Dilek Kurt, Hülya Bağdatlı Yılmaz

Institute of Pure and Applied Sciences, Marmara University, Istanbul, Türkiye
dilekyardim@marun.edu.tr

Department of Mathematics, Marmara University, Istanbul, Türkiye
hbagdatli@marmara.edu.tr

Abstract

This study aims to introduce the notion of almost pseudo $\mathcal{B}$-symmetric manifolds and to investigate their geometric structure under certain conditions. First, almost pseudo $\mathcal{B}$-symmetric Lorentzian manifolds endowed with a special type of semi-symmetric metric ρ-connection $\widetilde{\nabla}$ are considered. Then, the compatibility conditions involving the $\mathcal{B}$ tensor are examined and explicit geometric criteria governing these relations are obtained. As a consequence, several curvature properties of such manifolds are derived. Finally, the obtained results provide new characterizations of almost pseudo $\mathcal{B}$-symmetric Lorentzian manifolds and contribute to their classification within the framework of generalized curvature conditions.

Keywords: $\mathcal{B}$ tensor, almost pseudo symmetric maniold, semi-symmetric connection, Codazzi type.
2020 Mathematics Subject Classification: 53B30, 53C25, 53C50.

References

[1] Y. J. Suh, C. A. Mantica, U. C. De, P. Pal, Pseudo B-symmetric manifolds, *Int. J. Geom. Methods Mod. Phys.* **14** (9) (2017), 1750119.

[2] H. Bağdatlı Yılmaz, On pseudo Z-symmetric Lorentzian manifolds admitting a type of semi-symmetric metric connection, *Anal. Math. Phys.* **13** (2023), 18.

[3] S. K. Chaubey, Y. J. Suh, U. C. De, Characterizations of Lorentzian manifolds admitting semisymmetric metric connection, *Anal. Math. Phys.* **10** (2020), 61.

Characterization of Plane Elastic Curves with Caputo Fractional Derivative

Bahar Abalı, Ahmet Yücesan

Graduate School of Natural and Applied Sciences, Süleyman Demirel University, Isparta, Türkiye, baharabali@hotmail.com

Department of Mathematics, Faculty of Engineering and Natural Sciences, Süleyman Demirel University, Isparta, Türkiye, ahmetyucesan@sdu.edu.tr

Abstract

Elastic curves are curves that minimize bending energy and are characterized by the classical elastica theory via the Euler–Lagrange equation, whose solutions are expressed in terms of Jacobi elliptic functions. In this study, elastic curves in the Euclidean plane are re-characterized using the Caputo fractional derivative operator instead of the ordinary derivative operator. The Caputo derivative is preferred in modeling physical systems due to its advantages such as the vanishing derivative of constants and the preservation of classical initial conditions. Using the fractional differential geometry approach, fractional arc-length parameter and fractional curvature definitions are employed. The fractional Euler–Lagrange equation is derived via fractional calculus of variations, and it is shown that the fractional curvature satisfies the classical elastic curve equation. Elastic curves are classified for different fractional derivative orders. While the standard elastic curve is obtained in the integer-order case, as the fractional order decreases, the curvatures weaken and approach linear geometry. As the fractional order approaches zero, elastic curves exhibit rigid rod behavior. This indicates that the fractional order can be interpreted as a parameter controlling the degree of material flexibility. These findings establish the Caputo fractional differential geometry approach as a natural generalization of classical elastica theory, offering a novel framework for modeling memory-dependent mechanical systems.

Keywords: Elastic curves, Caputo fractional derivative, Euler-Lagrange equations, Jacobi elliptic functions, fractional curvature.
2020 Mathematics Subject Classification: 26A33, 53A04.

References

[1] O. P. Agrawal, Formulation of Euler-Lagrange equations for fractional variational problems, *Journal of Mathematical Analysis and Applications* **272** (2002), 368-379.

[2] O. P. Agrawal, Generalized Euler-Lagrange equations and transversality conditions for FVPs in terms of the Caputo derivative, *Journal of Vibration and Control* **13** (2007), 1217-1237.

[3] R.L. Bagley, P.J. Torvik, A theoretical basis for the application of fractional calculus to viscoelasticity, *Journal of Rheology* **27** (1983), 201-210.

[4] D. Baleanu, J. J. Trujillo, A new method of finding the fractional Euler-Lagrange and Hamilton equations within Caputo fractional derivatives, *Communications in Nonlinear Science and Numerical Simulation* **15** (2010), 1111-1115.

[5] M. Barrios, G. Reyero, M. Tidball, Necessary conditions to a fractional variational problem, *Statistics, Optimization & Information Computing* **10** (2021), 425-442.

[6] G. Brunnett, A new characterization of plane elastica, *Mathematical Methods in Computer Aided Geometric Design II* (1992), 35-44.

[7] M. Caputo, Linear Models of dissipation whose Q is almost frequency independent-II, *Geophysical Journal International* **13** (1967), 529-539.

[8] A. A. Kilbas, H.M. Srivastava, J.J. Trujillo, *Theory and applications of fractional differential equations*, Elsevier, 2006.

[9] K. A. Lazopoulos, A. K. Lazopoulos, Fractional differential geometry of curves and surfaces, *Progress in Fractional Differentiation and Applications* **2** (2016), 169-186.

[10] I. Podlubny, *Fractional Differential Equations*, Academic Press, 1999.

[11] D. A. Singer, Lectures on elastic curves and rods, *AIP Conference Proceedings* **1002** (2008), 3-32.

[12] T. Yajima, H. Nagahama, Differential geometry of viscoelastic models with fractional-order derivatives, *Journal of Physics A: Mathematical and Theoretical* **43** (2010), 385207.

[13] T. Yajima, S. Oiwa, K. Yamasaki, Geometry of curves with fractional-order tangent vector and Frenet-Serret formulas, *Fractional Calculus and Applied Analysis* **21** (2018), 1493-1505.

[14] T. Yajima, K. Yamasaki, Geometry of surfaces with Caputo fractional derivatives and applications to incompressible two-dimensional flows, *Journal of Physics A: Mathematical and Theoretical* **45** (2012), 065201.

On 3-Parameter Generalized Quaternions and Dual Quaternions with Bihyperbolic Generalized Tribonacci Number Components

Zehra İşbilir, Nurten Gürses, Murat Tosun
Department of Mathematics, Düzce University, Düzce, Türkiye,
zehraisbilir@duzce.edu.tr
Department of Mathematics, Yildiz Technical University, Istanbul, Türkiye,
nbayrak@yildiz.edu.tr
Department of Mathematics, Sakarya University, Sakarya, Türkiye,
tosun@sakarya.edu.tr

Abstract

In this presentation, our main aim is to bring together the 3-parameter generalized quaternions (3PGQs), dual quaternions, and bihyperbolic generalized Tribonacci numbers (BGTNs). For this purpose, we introduce the 3PGQs with BGTNs components and dual quaternions with BGTNs components. Then, we obtain recurrence relation, Binet formula, generating function, some summation formulas, matrix equation, and some algebraic properties for these new number systems. Moreover, we mathematically model the constructed theory we examined in this study, inspired by the graph theoretical framework, by drawing graphs.

Keywords: 3-parameter generalized quaternions, bihyperbolic numbers, bihyperbolic generalized Tribonacci numbers, dual quaternions, generalized Tribonacci numbers.
2010 Mathematics Subject Classification: 05A15, 05C15, 11B37, 11R52, 11Y55, 15A66.

References

[1] M. Akyiğit, H. H. Kösal, M. Tosun, Fibonacci generalized quaternions, *Adv. Appl Clifford Algebr.* **24**(3) (2014), 631–641.

[2] A. Z. Azak, Some new identities with respect to bihyperbolic Fibonacci and Lucas numbers, *International Journal of Sciences: Basic and Applied Sciences* **60**(2) (2021), 14–37.

[3] M. Bilgin, S. Ersoy, Algebraic properties of bihyperbolic numbers, *Adv Appl. Clifford Algebr.* **30** (2020), Article Number: 13, 17 pages.

[4] D. Bród, A. Szynal-Liana, I. Włoch, Bihyperbolic numbers of the Fibonacci type and their idempotent representation, *Comment. Math. Univ. Carolinae* **62**(4) (2021), 409–416.

[5] D. Bród, A. Szynal-Liana, I. Włoch, On some combinatorial properties of bihyperbolic numbers of the Fibonacci type, *Math. Methods Appl. Sci.* **44**(6) (2021), 4607–4615.

[6] F. Catoni, D. Boccaletti, R. Cannata, V. Catoni, E. Nichelatti, P. Zampetti, The Mathematics of Minkowski Space-Time with an Introduction to Commutative Hypercomplex Numbers, Birkhäuser Verlag, Basel, Boston, Berlin, 2008.

[7] G. Cerda-Morales, Third-order Jacobsthal generalized quaternions, *J. Geom. Symmetry Phys.* **50** (2018), 11–27.

[8] G. Cerda-Morales, On a generalization for Tribonacci quaternions, *Mediterr. J. Math.* **14** (2017), Article Number: 239, 12 pages.

[9] G. Cerda-Morales, On the third-order Jacobsthal and third-order Jacobsthal–Lucas sequences and their matrix representations, *Mediterr. J. Math.* **16** (2019), Article Number: 32, 12 pages.

[10] G. Cerda-Morales, A note on modified third-order Jacobsthal numbers, *Proyecciones (Antofagasta)*, **39**(2) (2020), 409–420.

[11] G. Cerda-Morales, Identities for third order Jacobsthal quaternions, *Adv. Appl. Clifford Algebr.* **27** (2017), 1043–1053.

[12] G. Cerda-Morales, Dual third-order Jacobsthal quaternions, *Proyecciones (Antofagasta)* **37**(4) (2018), 731–747.

[13] J. L. Cereceda, Binet's formula for generalized Tribonacci numbers, *Int. J. Math. Educ. Sci. Technol.* **46**(8) (2015), 1235–1243.

[14] J. Cockle, On systems of algebra involving more than one imaginary; and on equations of the fifth degree, *Philosophical Magazine*, **35**(238) (1849), 434–437.

[15] Z. Deniz, Çizgelerde Yol-Eşleme ve Renklendirme, Süleyman Demirel University, Graduate School of Applied and Natural Sciences, M. Sc. Thesis, Isparta, Türkiye, 2012.

[16] Z. Deniz, Sectionable tournaments: their topology and coloring, *Order* **40**(3) (2023), 469–491.

[17] L. E. Dickson, On the theory of numbers and generalized quaternions, *Amer. J. Math.* **46** (1924), 1–16.

[18] Z. Ercan, S. Yüce, On properties of the dual quaternions, *Eur. J. Pure Appl.* **4**(2) (2011), 142–146.

[19] S. Ergezer, Properties of Some Generalized Bihyperbolic Numbers and Polynomials, Nevşehir Hacı Bektaş Veli University, Graduate School of Natural and Applied Sciences, M. Sc. Thesis, Nevşehir, Türkiye, 2024.

[20] M. Feinberg, Fibonacci–Tribonacci, *Fibonacci Q.* **1**(3) (1963), 71–74.

[21] C. Flaut, V. Shpakivskyi, On generalized Fibonacci quaternions and Fibonacci-Narayana quaternions, *Adv. Appl. Clifford Algebr.* **23**(3) (2013), 673–688.

[22] W. Gerdes, Generalized Tribonacci numbers and their convergent sequences, *Fibonacci Q.* **16**(3) (1978), 269–275.

[23] L. W. Griffiths, Generalized quaternion algebras and the theory of numbers, *Amer. J. Math.* **50**(2) (1928), 303–314.

[24] H. Günay, N. Taşkara, Some properties of Padovan quaternion, *Asian-Eur. J. Math.* **12**(06) (2019), 2040017.

[25] H. Günay, Quaternions and Applications of Some Generalized Third Order Sequences, Selçuk University, The Graduate School of Natural and Applied Science of Selçuk University, M. Sc. Thesis, Konya, Türkiye, 2019.

[26] N. Gürses, Z. İşbilir, An extended framework for bihyperbolic generalized Tribonacci numbers, *Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat.* **73**(3) (2024), 765–786.

[27] N. Gürses, Z. İşbilir, On the combined Jacobsthal-Padovan generalized quaternions, *Adv. Studies: Euro-Tbilisi Math. J.* **15**(2) (2022), 55–70.

[28] W. R. Hamilton, On quaternions; or on a new system of imaginaries in algebra, *Philos. Mag.* **25**(3) (1844), 489–495.

[29] W. R. Hamilton, Lectures on Quaternions, Hodges and Smith, Dublin, 1853.

[30] W. R. Hamilton, Elements of Quaternions, Chelsea Pub Com, New York, 1969.

[31] A. F. Horadam, Complex Fibonacci numbers and Fibonacci quaternions, *Am. Math. Mon.* **70**(3) (1963), 289–291.

[32] M. R. Iyer, Some results on Fibonacci quaternions, *Fibonacci Q.* **7**(2) (1969), 201–210.

[33] Z. İşbilir, N. Gürses, Padovan and Perrin generalized quaternions, *Math. Methods Appl. Sci.* **45**(18) (2022), 12060–12076.

[34] Z. İşbilir, N. Gürses, Pell-Padovan generalized quaternions, *Notes Number Theory Discrete Math.* **27**(1) (2021), 171–187.

[35] Z. İşbilir, N. Gürses, M. Tosun, 3-parameter generalized quaternions with generalized Tribonacci numbers components, *Filomat*, **39**(9) (2025), 3003–3027.

[36] Z. İşbilir, N. Gürses, Examination of generalized Tribonacci dual quaternions, *Acta et Commentationes Universitatis Tartuensis de Mathematica* **27**(2) (2023), 235–255.

[37] Z. İşbilir, N. Gürses, Padovan, Perrin and Pell-Padovan dual quaternions, *Turkish Journal of Mathematics and Computer Science* **15**(1) (2023), 125–144.

[38] M. Jafari, Y. Yaylı, Generalized quaternions and their algebraic properties, *Commun. Fac. Sci. Ank. Series. A1*, **64**(1) (2015), 15–27.

[39] D. Jarden, Recurring sequences, *Jerusalem: Riveon Lematematika*, (1966).

[40] D. Kalman, Generalized Fibonacci numbers by matrix methods, *Fibonacci Q.* **20**(1) (1982), 73–76.

[41] C. Kızılateş, P. Catarino, N. Tuğlu, On the bicomplex generalized Tribonacci quaternions, *Mathematics* **7**(1) (2019), 80, 8 pages.

[42] V. Majernik, Quaternion formulation of the Galilean space-time transformation, *Acta Physica Slovaca*, **56**(1) (2006), 9–14.

[43] H. Mortazaasl, M. Jafari, A study of semi-quaternions algebra in semi-Euclidean 4-space, *Mathematical Science and Applications E-Notes* **2**(1) (2013), 20–27.

[44] A. B. Mamagani, M. Jafari, On properties of generalized quaternion algebra, *J. Nov. Appl. Sci.* **2**(12) (2013), 683–689.

[45] S. Olariu, Complex Numbers in n-Dimensions, North-Holland Mathematics Studies, Amsterdam, 2002.

[46] H. Pottmann, J. Wallner, Computational Line Geometry, Springer-Verlag Berlin Heidelberg, New York, 2001.

[47] S. Rabinowitz, Algorithmic manipulation of third-order linear recurrences, *Fibonacci Q.* **34**(5) (1996), 447–464.

[48] D. Rochon, M. Shapiro, On algebraic properties of bicomplex and hyperbolic numbers, *An. Univ. Oradea Fasc. Mat.* **11**(71) (2004), 110.

[49] B. Rosenfeld, Geometry of Lie Groups, Kluwer Academic Publishers, 1997.

[50] D. Savin, C. Flaut, C. Ciobanu, Some properties of the symbol algebras, *Carpathian J. Math.* **25**(2) (2009), 239–245.

[51] G. Sobczyk, The hyperbolic number plane, *Coll. Math. J.* **26**(4) (1995), 268–280.

[52] A. G. Shannon, A. F. Horadam, Some properties of third-order recurrence relations, *Fibonacci Q.* **10**(2) (1972), 135–146.

[53] A. G. Shannon, Iterative formulas associated with generalized third order recurrence relations, *SIAM Journal on Applied Mathematics* **23**(3) (1972), 364–368.

[54] A. G. Shannon, C. K. Wong, Some properties of generalized third order Pell numbers, *Notes Number Theory Discrete Math.* **14**(4) (2008), 16–24.

[55] A. G. Shannon, A. F. Horadam, Generating functions for powers of third order recurrence sequences, *Duke Math. J.* **38**(4) (1971), 791–794.

[56] A. G. Shannon, P. G. Anderson, A. F. Horadam,Properties of Cordonnier, Perrin and Van der Laan numbers, *Int. J. Math. Educ. Sci. Technol.* **37**(7) (2006), 825–831.

[57] N. J. A Sloane, The online encyclopedia of integer sequences, (1964) `http://oeis.org/`.

[58] Y. Soykan, On the recurrence properties of generalized Tribonacci sequence, *Earthline J. Math. Sci.* **6**(2) (2021), 253–269.

[59] Y. Soykan, Summing formulas for generalized Tribonacci numbers, *Univers. J. Math. Appl.* **3**(1) (2020), 1–11.

[60] Y. Soykan, Generalized Tribonacci numbers: Summing formulas, *International Journal of Advances in Applied Mathematics and Mechanics* **7**(3) (2020), 57–76.

[61] Y. Soykan, Generalized Pell-Padovan numbers, *Asian J. Adv. Res.* **11**(2) (2020), 8–28.

[62] Y. Soykan, On sum formulas for generalized Tribonacci sequence, *Journal of Scientific Research and Reports* **26**(7) (2020), 27–52.

[63] Y. Soykan, On generalized third-order Pell numbers, *Asian J. Adv. Res.* **6**(1) (2019), 1–18.

[64] Y. Soykan, Generalized Grahaml numbers, *J. Adv. Math. Comput.* **35**(2) (2020), 42–57.

[65] Y. Soykan, On four special cases of generalized Tribonacci sequence: Tribonacci-Perrin, modified Tribonacci, modified Tribonacci-Lucas and adjusted Tribonacci-Lucas sequences, *Journal of Progressive Research in Mathematics* **16**(3) (2020), 3056–3084.

[66] Y. Soykan, On generalized Narayana numbers, *Int. J. Adv. Appl. Math. Mech.* **7**(3) (2020), 43–56.

[67] Y. Soykan, A study on generalized Jacobsthal-Padovan numbers, *J. Math. Sci.* **4**(2) (2020), 227–251.

[68] Y. Soykan, On generalized Padovan numbers, *Int. J. Adv. Appl. Math. and Mech.* **10**(4) (2023), 72–90.

[69] Y. Soykan, On generalized reverse 3-primes numbers, *Journal of Scientific Research and Reports* **26**(6) (2020), 1–20.

[70] Y. Soykan, A study on generalized (r, s, t)-numbers, *MathLAB Journal* **7** (2020), 101–129.

[71] T. D. Şentürk, 3-Parameter Generalized Quaternions, Institute of Science, Kastamonu University, PhD Thesis, 2020.

[72] T. D. Şentürk, Z. Ünal, 3-parameter generalized quaternions, *Comput. Methods Funct. Theory* **22**(3) (2022), 575–608.

[73] T. D. Şentürk, G. Bilgici, A. Daşdemir, Z. Ünal, A study on Horadam hybrid numbers, *Turk. J. Math.* **44**(4) (2020), Article 10, 1212–1221.

[74] D. Taşcı, Padovan and Pell-Padovan quaternions, *Journal of Science and Arts* **42**(1) (2018), 125–132.

[75] T. Unger, N. Markin, Quadratic forms and space-time block codes from generalized quaternion and biquaternion algebras, *IEEE Trans Inform Theory* **57**(9) (2011), 6148–6156.

[76] M. E. Waddill, Using matrix techniques to establish properties of a generalized Tribonacci sequence, *Applications of Fibonacci Numbers*, **4** (1991), 299–308.

[77] M. E. Waddill, L. Sacks, Another generalized Fibonacci sequence, *Fibonacci Q.* **5**(3) (1967), 209–222.

[78] D. B. West, Introduction to Graph Theory, Volume 2. Prentice Hall Upper Saddle River, 2001.

[79] C. C. Yalavigi, Properties of Tribonacci numbers, *Fibonacci Q.* **10**(3) (1972), 231–246.

[80] S. Yüce, Sayılar ve Geometri, Pegem Academy Publishing, Ankara, 2020.

[81] S. Yüce, F. Torunbalcı Aydın, A new aspect of dual Fibonacci quaternions, *Adv. Appl. Clifford Algebr.* **26** (2016), 873–884.

[82] S. Yüce, F. Torunbalcı Aydın, Generalized dual Fibonacci quaternion, *Appl. Math. E-Notes* **16**(309) (2016), 276–289.

Stationary Acceleration Curves in SE(3) Under the Combescure Transformation

Kadir İçaçan, Yusuf Yaylı

Department of Computer Technologies, Adiyaman University, Adiyaman, Türkiye, e-mail: kicacan@adiyaman.edu.tr

Department of Mathematics, Faculty of Science, Ankara University, Ankara, Türkiye

Abstract

In this study, a general curve pair model defined by using the Frenet frame for unit-speed curves in the three-dimensional Euclidean space is considered. In this model, the rigid body motions corresponding to the initial curve and the associated general curve pair are defined on the Special Euclidean Group $SE(3)$, and the conditions under which these motions have stationary acceleration are investigated. For this purpose, the velocity, acceleration, and stationary acceleration matrices corresponding to the rigid body motions of the initial curve and the general curve pair are obtained, and the conditions for preserving stationary acceleration motions are determined. Subsequently, the Combescure transformation is considered as a special case of the general model. In this case, necessary and sufficient conditions are obtained for the rigid body motion corresponding to the curve obtained under the Combescure transformation to be a stationary acceleration motion. Furthermore, it is shown that, since the initial curve and the Combescure curve share the same Frenet frame, the rotational part of the two rigid body motions remains unchanged, while the effect of the transformation appears only in the translational component. Finally, the obtained theoretical results are verified by means of a unit-speed circular helix example.

Keywords: Combescure transformation; stationary acceleration curves; stationary acceleration motions; Frenet frame; rigid body motions; Special Euclidean Group; $SE(3)$.

2020 Mathematics Subject Classification: 53A17, 53A04, 70E17.

References

[1] O. Keçilioğlu and K. İlarslan, Non-null Space Curves Related by a Transformation of Combescure, *Filomat* **39**(25) (2025), 8789–8800.

[2] J. M. Selig, *Geometric Fundamentals of Robotics*, 2nd ed., Springer, New York, 2005.

[3] J. M. Selig, Curves of Stationary Acceleration in $SE(3)$, *IMA Journal of Mathematical Control and Information* **24**(3) (2007), 271–281.

List of Participants

1	Abdullah Mağden	Bursa Technical University	TÜRKİYE
2	Ahmet Mollaoğulları	Çanakkale Onsekiz Mart University	TÜRKİYE
3	Ahmet Yücesan	Süleyman Demirel University	TÜRKİYE
4	Ali Çakmak	Ağrı İbrahim Çeçen University	TÜRKİYE
5	Alev Kelleci Akbay	Iskenderun Technical University	TÜRKİYE
6	Arif Salimov	Baku State University	AZERBAIJAN
7	Arzu Sürekçi	Sakarya University of Applied Sciences	TÜRKİYE
8	Aslı Aybüke Akdemir	Gaziantep University	TÜRKİYE
9	Atakan Tuğkan Yakut	Niğde Ömer Halisdemir Univsersity	TÜRKİYE
10	Aydın Gezer	Atatürk University	TÜRKİYE
11	Aykut Kayhan	Maltepe University	TÜRKİYE
12	Aykut Toplama	Kilis 7 Aralık University	TÜRKİYE
13	Ayşe Yılmaz Ceylan	Akdeniz University	TÜRKİYE
14	Ayşe Zeynep Azak	Sakarya University	TÜRKİYE
15	Ayşe Yavuz	Necmettin Erbakan University	TÜRKİYE
16	Aziz Yayla	Selçuk University	TÜRKİYE
17	Bahar Doğan Yazıcı	Bilecik Şeyh Edebali University	TÜRKİYE
18	Başak Yağbasan	Eskişehir Osmangazi University	TÜRKİYE
19	Bedia Merih Özçetin Vatansever	Istanbul Gelişim University	TÜRKİYE
20	Berke Kaya		
21	Betül Bulca Sokur	Bursa Uludağ University	TÜRKİYE
22	Burak Şahiner	Manisa Celal Bayar University	TÜRKİYE
23	Burçin Doğan	Malatya Turgut Özal University	TÜRKİYE
24	Buşra Aktaş	Kırıkkale University	TÜRKİYE
25	Burçin Saltık Baek	Erciyes University	TÜRKİYE
26	Bülent Karakaş	Bartın University	TÜRKİYE
27	Büşra Ay		
28	Cansel Aycan Özdemir	Pamukkale University	TÜRKİYE
29	Cengizhan Murathan	Bursa Uludağ University	TÜRKİYE
30	Cihan Özgür	İzmir Democracy University	TÜRKİYE
31	Cumali Ekici	Eskişehir Osmangazi University	TÜRKİYE
32	Çetin CAMCI	Çanakkale Onsekiz Mart University	TÜRKİYE
33	Dilara Kocatürk	Süleyman Demirel University	TÜRKİYE
34	Dilek Demirhan	Pamukkale University	TÜRKİYE
35	Dilek Kurt	Marmara University	TÜRKİYE
36	Doğukan Kesik		
37	Erol Kılıç	İnönü University	TÜRKİYE
38	Erol Yaşar	Mersin University	TÜRKİYE

39	Esin Akgün	Balıkesir University	TÜRKİYE
40	Esra Damar	Hitit University	TÜRKİYE
41	Elvan Korkmaz	Giresun University	TÜRKİYE
42	Eylül Ayyıldız	Süleyman Demirel University	TÜRKİYE
43	Faik Duran	Çankırı Karatekin University	TÜRKİYE
44	Fatma Almaz	Batman University	TÜRKİYE
45	Ferdağ Kahraman Aksoyak	Kırşehir Ahi Evren University	TÜRKİYE
46	Fereidoun Sabetghadam	Islamic Azad University	IRAN
47	Filiz Ocak	Karadeniz Technical University	TÜRKİYE
48	Gizem Güzelkardeşler	Karadeniz Technical University	TÜRKİYE
49	Gökhan Arıkan	İnönü University	TÜRKİYE
50	Gözde Özkan Tükel	Isparta University of Applied Sciences	TÜRKİYE
51	Gül Uğur Kaymanlı	Çankırı Karatekin University	TÜRKİYE
52	Gülay Koru Yücekaya	Gazi University	TÜRKİYE
53	Gülden Kapusuz	Isparta University of Applied Sciences	TÜRKİYE
54	Gülsüm Yüca	Aksaray University	TÜRKİYE
55	Gülsümnur Coşkun	Süleyman Demirel University	TÜRKİYE
56	Günay Öztürk	İzmir Democracy University	TÜRKİYE
57	Habil Fattayev	Baku State University	
58	Hacı Bayram Karadağ	İnönü University	TÜRKİYE
59	Hakan Mete Taştan	İstanbul University	TÜRKİYE
60	Halil İbrahim Dönmez	Süleyman Demirel University	TÜRKİYE
61	Hanim Gonca Peştere		
62	Hatice Çetinkaya	Selçuk University	TÜRKİYE
63	Hatice Sena Savcu	Süleyman Demirel University	TÜRKİYE
64	Hatice Tozak	MEB Denizli Provincial Directorate of National Education	TÜRKİYE
65	Hidayet Hüda Kösal	Sakarya University	TÜRKİYE
66	Hülya Aytimur	Balıkesir University	TÜRKİYE
67	Hülya Gün Bozok	Osmaniye Korkut Ata University	TÜRKİYE
68	İbrahim Günaltılı	Eskişehir Osmangazi University	TÜRKİYE
69	İbrahim Gürgil	Uşak University	TÜRKİYE
70	İlkay Güven	Gaziantep University	TÜRKİYE
71	İsmet Gölgeleyen	Zonguldak Bülent Ecevit University	TÜRKİYE
72	İrem Küpeli Erken	Bursa Technical University	TÜRKİYE
73	İrem Üstüner	İnönü University	TÜRKİYE
74	Kadri Arslan	Bursa Technical University	TÜRKİYE
75	Kazım İlarslan	Kırıkkale University	TÜRKİYE
76	Kemalettin Can Deniz	Süleyman Demirel University	TÜRKİYE
77	Kevser Soytürk	Tekirdağ Namık Kemal University	TÜRKİYE
78	Medine Bakraç	Yildiz Technical University	TÜRKİYE

79	Mehrican Gidal		
80	Mehmet Bektaş	Fırat University	TÜRKİYE
81	Mehmet Çaldır	Çukurova University	TÜRKİYE
82	Mehmet Duru	Akdeniz University	TÜRKİYE
83	Melek Erdoğdu	Necmettin Erbakan University	TÜRKİYE
84	Meltem Karaismailoğlu	Kırşehir Ahi Evran University	TÜRKİYE
85	Muhittin Evren Aydın	Fırat University	TÜRKİYE
86	Murat Altunbaş	Tekirdağ Namık Kemal Üniversitesi	TÜRKİYE
87	Murat Babaarslan	Yozgat Bozok University	TÜRKİYE
88	Murat Kemal Karacan	Uşak University	TÜRKİYE
89	Murat Tosun	Sakarya University	TÜRKİYE
90	Mustafa Dede	Kilis 7 Aralık University	TÜRKİYE
91	Mustafa Özdemir	Akdeniz University	TÜRKİYE
92	Mustafa Özkan	Bursa Technical University	TÜRKİYE
93	Mustafa Özkan	Gazi University	TÜRKİYE
94	Neslihan Yıldırım	Gazi University	TÜRKİYE
95	Neriman Acar	Kırşehir Ahi Evran University	TÜRKİYE
96	Nihat Ayyıldız	Süleyman Demirel University	TÜRKİYE
97	Nuran Yüksel	Erciyes University	TÜRKİYE
98	Nurdan Oğraş	Erciyes University	TÜRKİYE
99	Nurettin Cenk Turgay	Istanbul Technical University	TÜRKİYE
100	Nursemin Çavdar	Tekirdağ Namık Kemal Üniversitesi	TÜRKİYE
101	Olgun Durmaz	Erzurum Technical University	TÜRKİYE
102	Onur Kaya	Manisa Celal Bayar University	TÜRKİYE
103	Osman Alemdar	Karadeniz Technical University	TÜRKİYE
104	Özcan Gelişgen	Eskişehir Osmangazi University	TÜRKİYE
105	Şaban Güvenç	Balıkesir University	TÜRKİYE
106	Semra Kaya	Uşak University	TÜRKİYE
107	Semra Zeren	Sivas University of Science and Technology	TÜRKİYE
108	Serdar Soylu	Giresun University	TÜRKİYE
109	Serpil Ünal	Uşak University	TÜRKİYE
110	Sezin Aykurt Sepet	Kırşehir Ahi Evran University	TÜRKİYE
111	Sırma Zeynep Alparslan Gök	Süleyman Demirel University	TÜRKİYE
112	Sibel Pasalı Atmaca	Muğla Sıtkı Koçman University	TÜRKİYE
113	Sibel Turanlı	Erzurum Technical University	TÜRKİYE
114	Sinem Güler	Istanbul Sabahattin Zaim University	TÜRKİYE
115	Stefan Pickl	Bundeswehr Munich University	GERMANY
116	Şemsi Eken Meriç	Mersin University	TÜRKİYE
117	Şenay Baydaş	Van Yüzüncü Yıl University	TÜRKİYE
118	Şerife Nur Bozdağ	Ege University	TÜRKİYE

119	Şifa Aygün	Tekirdağ Namık Kemal University	TÜRKİYE
120	Tayfun Toprak	Karadeniz Technical University	TÜRKİYE
121	Tugay Türkmen	Uşak University	TÜRKİYE
122	Tuğba Gençer	Ağrı İbrahim Çeçen University	TÜRKİYE
123	Tuğba Tamirci	Gazi University	TÜRKİYE
124	Tülay Erişir	Erzincan Binali Yıldırım University	TÜRKİYE
125	Vildan Özdemir	Selçuk University	TÜRKİYE
126	Yağmur Karabıyık	Süleyman Demirel University	TÜRKİYE
127	Yasemin Soylu	Giresun University	TÜRKİYE
128	Zehra İşbilir	Düzce University	TÜRKİYE
129	Zeynep İsabeyoğlu	Erzincan Binali Yıldırım University	TÜRKİYE
130	Zülal Derin Yaqub	Istanbul Technical University	TÜRKİYE
131	Kadir İçaçan	Adiyaman University	TÜRKİYE
132	Yusuf Yaylı	Ankara University	TÜRKİYE
133	Tunahan Turhan	Süleyman Demirel University	TÜRKİYE

ACADEMIC SPONSORS

Süleyman Demirel University

Sakarya University

Sakarya University of Applied Sciences

Anadolu University

Baku State University

Geometers Society