BULANIK SAYILARIN ÇİFT DİZİSİNİN KABA İSTATİSTİKSEL YAKINLAŞMASI

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ÖZET

$\mathcal{A}$ kümesi, $\mathbb{N}$ pozitif tam sayıların bir alt kümesi olmak üzere bu kümenin doğal yoğunluğu $\delta(\mathcal{A}) = \lim_{n\to\infty} \frac{|\{a\leq n: a\in\mathcal{A}\}|}{n}$ şeklinde tanımlanır. Burada $|\{a \leq n: a \in \mathcal{A}\}|$ kümesi $\mathcal{A}$ nin n'den büyük olmayan elemanlarının sayısını göstermektedir. Eğer $\forall\, \varepsilon > 0$ için $\delta(\{\gamma \leq n: |x_\gamma - \mathcal{L}| \geq \varepsilon\}) = 0$ ise (x_γ) dizisi $\mathcal{L}$ sayısına istatistiksel yakınsaktır denir. Bu kavram ölçü teorisi, toplanabilme teorisi ve optimizasyon teorisi gibi matematiğin temel alanlarıyla olan ilişkisi nedeniyle uzun yıllardır bir çok matematikçi tarafından ilgiyle çalışılmaktadır. Son zamanlarda, $n-$ boyutlu reel uzayda terimleri kesin olarak hesaplanamayan istatistiksel yakınsak bir $y = (y_\gamma)$ dizisi yerine $\|x_\gamma - y_\gamma\| \leq r$ şartını sağlayan ve terimleri bilinen $x = (x_\gamma)$ yaklaşım dizisi ile işlem yapmanın kolaylık sağladığı görülmüştür. Böylelikle kaba istatistiksel yakınsaklık olarak adlandırılan yeni bir kavram ortaya çıkmıştır. Bu yaklaşım dizisi istatistiksel yakınsak değildir ancak r kabalık derecesi ile kaba istatistiksel yakınsaktır denir. Bu çalışmada biz, vektör uzay özelliklerine sahip olmadığı için normlu uzay kategorisinde olmayan ve toplanabilme teorisinde birçok uygulama alanına sahip bulanık sayılar uzayındaki çift dizilerin kaba istatistiksel yakınsaklığını tanımlayacağız. Genel olarak, kabalık derecesi $r > 0$ olduğunda dizinin kaba istatistiksel limiti tek olmayacağından kaba istatistiksel limit kümesi oluşturarak bu kümenin özelliklerini vereceğiz. Son olarak, çift bulanık sayı dizileri için istatistiksel yakınsaklık ve kaba istatistiksel yakınsaklık arasındaki ilişkiyi inceleyeceğiz.

Anahtar Kelimeler: Çift dizi, bulanık sayı, istatistiksel yakınsaklık, kaba istatistiksel yakınsaklık.

ROUGH STATISTICAL CONVERGENCE OF DOUBLE SEQUENCES OF FUZZY NUMBERS

ABSTRACT

The natural density of a subset $\mathcal{A}$ of positive integers $\mathbb{N}$ is defined as $\delta(\mathcal{A}) = \lim_{n\to\infty} \frac{|\{a\leq n: a\in\mathcal{A}\}|}{n}$. Here, the set $|\{a \leq n: a \in \mathcal{A}\}|$ indicates the cardinality of $\mathcal{A}$ not greater than n. The sequence (x_γ) is said to be statistically convergent to $\mathcal{L}$ if for $\forall\, \varepsilon > 0$, $\delta(\{\gamma \leq n: |x_\gamma - \mathcal{L}| \geq \varepsilon\}) = 0$. This concept has been studied with interest by many mathematicians for many years due to its relevance to fundamental areas of mathematics such as measure theory, summability theory, number theory and optimization theory. Recently, instead of a statistically convergent sequence $y = (y_\gamma)$ whose terms in n-dimensional real space cannot be exactly calculated (or measured), it has been found that it is easier to operate with an approximation sequence $x = (x_\gamma)$ whose terms are known such that $\|x_\gamma - y_\gamma\| \leq r$. Thus, a new concept called rough statistical convergence emerged. This approximation sequence is not statistically convergent, but it is said to be rough statistically convergent with r.

In this work, we will study this notion for double sequences in the space of fuzzy numbers, which is not in the category of normed space because it does not have the properties of a vector space and has many applications in summability theory. If $r > 0$, we will construct r - statistical limit set and give some properties of this set. Finally, we will examine the relationship between statistical convergence and its rough analogue for double sequences of fuzzy numbers.
Keywords: Double sequence, fuzzy number, statistical convergence, rough statistical convergence.

1. INTRODUCTION

In finite-dimensional normed spaces, the concept of rough convergence was defined by Phu in 2001 [8]. In 2008, Aytar [3] generalized the idea of rough convergence to n-dimensional real spaces using the concept of natural density. Akçay and Aytar [1] investigated the rough convergence of sequences in the space of fuzzy numbers, which do not have vector space properties and do not belong to the normed space category. Debnath and Rakshit [5] worked on the rough statistical convergence of these sequences. Due to their applications in various areas, double sequences have attracted the interest of many mathematicians. For example, Dündar and Çakan [6] studied the rough convergence of these sequences, while Kişi and Ünal [7] worked on the rough statistical convergence of double-difference sequences in finite-dimensional spaces. Additionally, Savaş and Mursaleen [10] studied the statistical convergence of double sequences of fuzzy number which given by Savaş [9], Babaarslan and Tuncer [4] studied on rough convergence for these sequences.

In this paper, firstly, the rough statistical convergence and the rough statistical limit set of double sequences of fuzzy numbers are defined. Moreover, the geometric and topological properties of this set are provided and the relationship between statistical convergence and rough statistical convergence is discussed.

2. PRELIMINARIES

First, a reminder of some basic notions in fuzzy number theory.

Definition 2.1. A fuzzy number is a function $X:\mathbb{R}\to[0,1]$ such that it is normal, fuzzy convex, upper semi-continuous and the closure of the set $\{x\in\mathbb{R} : X(x)>0\}$ is compact. The β −level set of this number is defined by $X^{\beta}:=\{x\in\mathbb{R} : X(x)\geq\beta\}=\left[\underline{X}^{\beta},\overline{X}^{\beta}\right]$ for each $\beta\in(0,1]$.

The set of all fuzzy numbers is denoted by $L(\mathbb{R})$. Also, $(L(\mathbb{R}),\bar{d})$ is a complete metric space with the metric $\bar{d}(X,Y)=\sup_{0\leq\beta\leq1}\max\left\{\left|\underline{X}^{\beta}-\underline{Y}^{\beta}\right|,\left|\overline{X}^{\beta}-\overline{Y}^{\beta}\right|\right\}$.

Definition 2.2. [9] A double sequence $X=(X_{\gamma\varsigma})$ of fuzzy numbers is a function X from $\mathbb{R}\times\mathbb{R}$ into $L(\mathbb{R})$. $X_{\gamma\varsigma}$ is the function's value at point (γ,ς).

Throughout the paper, $X=(X_{\gamma\varsigma})$ denotes a double sequence of fuzzy numbers and $\tilde{X}$ denotes a fuzzy number.

Definition 2.3. [9] A sequence $X=(X_{\gamma\varsigma})$ is said to be convergent in Pringsheim's sense to $\tilde{X}$, denoted by $P-limX=\tilde{X}$, if $(X_{\gamma\varsigma})$ tends to $\tilde{X}$ as $\gamma\to\infty$, $\varsigma\to\infty$.

Definition 2.4. [9] A sequence $X=(X_{\gamma\varsigma})$ is said to be Cauchy, if for every $\varepsilon>0$, there exists $N_0\in\mathbb{R}$ provided that $\bar{d}(X_{\gamma\varsigma},\tilde{X})<\varepsilon$, whenever $\gamma\geq j\geq N_0$ and $\varsigma\geq k\geq N_0$.

Definition 2.5. [10] Let $\mathcal{M} \subset \mathbb{N} \times \mathbb{N}$ and $\mathcal{M}(\gamma, \varsigma)$ be the numbers of (j,k) in $\mathcal{M}$ provided that $j \leq \gamma$ and $k \leq \varsigma$. Then, the limit

$$\delta_2(\mathcal{M}) = \lim_{\gamma,\varsigma} \frac{\mathcal{M}(\gamma,\varsigma)}{\gamma\varsigma}$$

is called double natural density of $\mathcal{M}$.

Definition 2.6. [10] A sequence $X = (X_{\gamma\varsigma})$ is said to be statistically convergent to $\tilde{X}$, denoted by $St_2 - limX_{\gamma\varsigma} = \tilde{X}$, if for every $\varepsilon > 0$,

$\lim_{n,m} \frac{1}{nm} \left|\{(\gamma,\varsigma): \gamma \leq n \text{ and } \varsigma \leq m: \bar{d}(X_{\gamma\varsigma}, \tilde{X}) \geq \varepsilon\}\right| = 0.$

Definition 2.7. [4] A sequence $X = (X_{\gamma\varsigma})$ is said to be rough convergent in Pringsheim's sense to $\tilde{X}$, denoted by $X_{\gamma\varsigma} \xrightarrow{r} \tilde{X}$, if for every $\varepsilon > 0$, there exists an integer i_ε such that

$$\bar{d}(X_{\gamma\varsigma}, \tilde{X}) < r + \varepsilon,$$

whenever $\gamma, \varsigma \geq i_\varepsilon$.

Here, $\tilde{X}$ is called r -limit point and r is called roughness degree. If $r = 0$, the rough convergence reduces the convergence in Pringsheim's sense.

Definition 2.8. [4] A sequence $X = (X_{\gamma\varsigma})$ is said to be rough Cauchy, if for every $\varepsilon > 0$, there exists an integer i_ε such that

$$\bar{d}(X_{\gamma\varsigma}, X_{kl}) < \rho + \varepsilon$$

whenever $k \geq \gamma \geq i_\varepsilon$, $l \geq \varsigma \geq i_\varepsilon$.

3. MAIN RESULTS

Definition 3.1. Let $\varepsilon \succ 0$. A sequence $X = (X_{\gamma\varsigma})$ is said to be rough statistically convergent to $\tilde{X}$, written as $X_{\gamma\varsigma} \xrightarrow{r-St_2} \tilde{X}$, if for every $\varepsilon > 0$,

$\lim_{n,m} \frac{1}{nm} \left|\{(\gamma,\varsigma): \gamma \leq n \text{ and } \varsigma \leq m: \bar{d}(X_{\gamma\varsigma}, \tilde{X}) \geq r + \varepsilon\}\right| = 0.$

Also, r is called the roughness degree. For $r = 0$, then we reduce the concept of statistical convergence in Pringsheim's sense.

Just as the concept of rough statistical convergence applies to a sequence of fuzzy numbers, we can interpret our idea in a similar manner as follows:

Suppose that the sequence $(Y_{\gamma\varsigma})$ is statistically convergent to $\tilde{X}$. However, we cannot precisely determine the elements of $(Y_{\gamma\varsigma})$. If calculating with the elements of $(Y_{\gamma\varsigma})$ proves to be challenging, we can instead obtain an approximated sequence of triangular fuzzy numbers $(X_{\gamma\varsigma})$ satisfying the condition $\bar{d}(X_{\gamma\varsigma}, Y_{\gamma\varsigma}) \leq r$. If it is difficult to calculate with the elements of $(Y_{\gamma\varsigma})$, then we obtain a triangular fuzzy number sequences $(X_{\gamma\varsigma})$ (approximated sequence) satisfying $\bar{d}(X_{\gamma\varsigma}, Y_{\gamma\varsigma}) \leq r$. This allows us to use an approximate value of $(Y_{\gamma\varsigma})$ for simpler calculations. In this case, $(X_{\gamma\varsigma})$ may not statistically convergent, but it serves as the inclusion

$$\{(\gamma,\varsigma): \gamma \leq n \text{ ve } \varsigma \leq m: \bar{d}(Y_{\gamma\varsigma}, \tilde{X}) \geq \varepsilon\} \supseteq \{(\gamma,\varsigma): \gamma \leq n \text{ ve } \varsigma \leq m: \bar{d}(X_{\gamma\varsigma}, \tilde{X}) \geq r + \varepsilon\}$$

holds and from our assumption we get the sequence $(X_{\gamma\varsigma})$ is rough statistically convergent to $\tilde{X}$.

The rough statistical limit of a double sequence may not have a unique value for a given r. To address this, we define the r −statistical limit set as follows:

$St_2 - LIM^r X_{\gamma\varsigma} := \{\tilde{X} \in L(\mathbb{R}^n): X_{\gamma\varsigma} \xrightarrow{r-St_2} \tilde{X}\}.$

Definition 3.2. A sequence $X = (X_{\gamma\varsigma})$ is said to be rough statistically Cauchy, if for every $\varepsilon > 0$, there exists an integer i_ε such that
$\lim_{n,m} \frac{1}{nm} \left|\{(\gamma,\varsigma): \gamma \le n \text{ and } \varsigma \le m: \bar{d}(X_{\gamma\varsigma}, X_{kl}) \ge r + \varepsilon\}\right| = 0,$
whenever $k \ge n \ge i_\varepsilon,\ l \ge m \ge i_\varepsilon$.

Teorem 3.1. If $X = (X_{\gamma\varsigma})$ is r- statistically convergent to $\tilde{X}$, then it is $2r$- statistically Cauchy.

Proof. Let $\varepsilon > 0$. Since $X_{\gamma\varsigma} \xrightarrow{r-St_2} \tilde{X}$, then we have
$\lim_{n,m} \frac{1}{nm} \left|\{(\gamma,\varsigma): \gamma \le n \text{ and } \varsigma \le m: \bar{d}(X_{\gamma\varsigma}, \tilde{X}) \ge r + \varepsilon/2\}\right| = 0,$
that is, for almost all $(\gamma,\varsigma) \in \mathbb{N} \times \mathbb{N}$, we write $\bar{d}(X_{\gamma\varsigma}, \tilde{X}) < r + \varepsilon/2$.
Hence, for almost all (γ,ς), we get

$$\bar{d}(X_{\gamma\varsigma}, X_{kl}) \le \bar{d}(X_{\gamma\varsigma}, \tilde{X}) + \bar{d}(X_{kl}, \tilde{X}) < r + \frac{\varepsilon}{2} + r + \frac{\varepsilon}{2} = 2r + \varepsilon.$$

This shows that $X = (X_{\gamma\varsigma})$ is $2r$- statistically Cauchy.

Theorem 3.2. The diameter of $\left(St_2 - LIM^r X_{\gamma\varsigma}\right) \le 2r$.

Proof. Suppose that $diam\left(St_2 - LIM^r X_{\gamma\varsigma}\right) > 2r$. Then, there exists $\mathcal{Y}, \mathcal{V} \in St_2 - LIM^r X_{\gamma\varsigma}$ provided that $\bar{d}(\mathcal{Y}, \mathcal{V}) > 2r$. Let $\varepsilon \in \left(0, \frac{\bar{d}(\mathcal{Y},\mathcal{V})}{2} - r\right)$.
Since $\mathcal{Y}, \mathcal{V} \in St_2 - LIM^r X_{\gamma\varsigma}$, we have $\delta_2(\mathcal{A}_1(\varepsilon)) = 0$ and $\delta_2(\mathcal{A}_2(\varepsilon)) = 0$ for $\forall \varepsilon > 0$, where

$$\mathcal{A}_1(\varepsilon) := \{(\gamma,\varsigma) : \bar{d}(X_{\gamma\varsigma}, \mathcal{Y}) \ge r + \varepsilon\}$$

and
$\mathcal{A}_2(\varepsilon) := \{(\gamma,\varsigma) : \bar{d}(X_{\gamma\varsigma}, \mathcal{V}) \ge r + \varepsilon\}.$
Thus, for all $(\gamma,\varsigma) \in \mathcal{A}_1^c(\varepsilon) \cap \mathcal{A}_2^c(\varepsilon)$, we get

$$\bar{d}(\mathcal{Y}, \mathcal{V}) \le \bar{d}(X_{\gamma\varsigma}, \mathcal{Y}) + \bar{d}(X_{\gamma\varsigma}, \mathcal{V}) < 2(r + \varepsilon) < 2\left(r + \frac{\bar{d}(\mathcal{Y}, \mathcal{V})}{2} - r\right) = \bar{d}(\mathcal{Y}, \mathcal{V})$$

which is a contradiction.

Theorem 3. 3. The set $St_2 - LIM^r(X_{\gamma\varsigma})$ is closed.
Proof. If $St_2 - LIM^r(X_{\gamma\varsigma}) = \emptyset$, it is obvious. Let $St_2 - LIM^r(X_{\gamma\varsigma}) \ne \emptyset$ and $(Y_{\gamma\varsigma})$ be a sequence in $St_2 - LIM^r(X_{\gamma\varsigma})$ which $Y_{\gamma\varsigma} \to \tilde{Y}$ as $\gamma, \varsigma \to \infty$.
$\varepsilon > 0$ be given. Since $Y_{\gamma\varsigma} \to \tilde{Y}$, there exists $\gamma_{\frac{\varepsilon}{2}}$ and $\varsigma_{\frac{\varepsilon}{2}}$ such that $\bar{d}(Y_{\gamma\varsigma}, \tilde{Y}) < \frac{\varepsilon}{2}$ for all $\gamma > \gamma_{\frac{\varepsilon}{2}}$ and $\varsigma > \varsigma_{\frac{\varepsilon}{2}}$.
Now, we choose a (γ_0, ς_0) such that $\gamma_0 > \gamma_{\frac{\varepsilon}{2}}, \varsigma_0 > \varsigma_{\frac{\varepsilon}{2}}$. Then, we write
$\bar{d}(Y_{\gamma_0\varsigma_0}, \tilde{Y}) < \frac{\varepsilon}{2}$.
Also, since $(Y_{\gamma\varsigma}) \subseteq St_2 - LIM^r(X_{\gamma\varsigma})$, we have $Y_{\gamma_0\varsigma_0} \subseteq St_2 - LIM^r(X_{\gamma\varsigma})$, that is

$$\lim_{n,m} \frac{1}{nm} \left|\left\{(\gamma,\varsigma): \bar{d}(X_{\gamma\varsigma}, Y_{\gamma_0\varsigma_0}) \ge r + \frac{\varepsilon}{2}\right\}\right| = 0.$$

When we choose an arbitrary $(p,q) \in \left\{(\gamma,\varsigma): \bar{d}(X_{\gamma\varsigma}, Y_{\gamma_0\varsigma_0}) < r + \frac{\varepsilon}{2}\right\}$, then we get $\bar{d}(X_{pq}, Y_{\gamma_0\varsigma_0}) < r + \frac{\varepsilon}{2}$.

Hence, we can write the following inequality

$$\bar{d}(X_{pq}, \tilde{Y}) \leq \bar{d}(X_{pq}, Y_{\gamma_0\varsigma_0}) + \bar{d}(Y_{\gamma_0\varsigma_0}, \tilde{Y}) < r + \varepsilon.$$

Thus, the conclusion

$$\{(\gamma,\varsigma): \bar{d}(X_{\gamma\varsigma}, \tilde{Y}) < r+\varepsilon\} \supseteq \left\{(\gamma,\varsigma): \bar{d}(X_{\gamma\varsigma}, Y_{\gamma_0\varsigma_0}) < r + \frac{\varepsilon}{2}\right\}$$

is holds.

Since $\delta_2\left(\left\{(\gamma,\varsigma): \bar{d}(X_{\gamma\varsigma}, Y_{\gamma_0\varsigma_0}) < r + \frac{\varepsilon}{2}\right\}\right) = 1$, then we get

$$\delta_2(\{(\gamma,\varsigma) \in \mathbb{N} \times \mathbb{N}: \bar{d}(X_{\gamma\varsigma}, \tilde{Y}) < r+\varepsilon\}) = 1.$$

This shows that $\tilde{Y} \in St_2 - LIM^r(X_{\gamma\varsigma})$.

Theorem 3. 4. If $St_2 - limX_{\gamma\varsigma} = \tilde{X}$ ($X_{\eta\varsigma}$), then

$$St_2 - LIM^r(X_{\gamma\varsigma}) = \overline{B_r}(\tilde{X}) := \{\mu \in L(\mathbb{R}^n): \bar{d}(\mu, \tilde{X}) \leq r\}.$$

Proof. . Let $\mathcal{A}_1 := \{(\gamma,\varsigma) : \bar{d}(X_{\gamma\varsigma}, Y) \geq \varepsilon\}$. From our assumption $\delta_2(\mathcal{A}_1) = 0$.

Let we take $Y \in \overline{B_r}(\tilde{X})$.

For all $(\gamma,\varsigma) \in \mathcal{A}_1^C$, we get

$$\bar{d}(X_{\gamma\varsigma}, Y) \leq \bar{d}(X_{\gamma\varsigma}, \tilde{X}) + \bar{d}(\tilde{X}, Y) < \varepsilon + r$$

which shows that $Y \in St_2 - LIM^r(X_{\gamma\varsigma})$.

Let $\mathcal{A}_2 := \{(\gamma,\varsigma) : \bar{d}(X_{\gamma\varsigma}, Y) \geq r + \varepsilon\}$. Now, we take an arbitrary $Y \in St_2 - LIM^r(X_{\gamma\varsigma})$. So, we write $\delta_2(\mathcal{A}_2) = 0$. Also, from our assumption, we have $\delta_2(\mathcal{A}_1) = 0$.

For all $(\gamma,\varsigma) \in \mathcal{A}_1^C \cap \mathcal{A}_2^C$, we get

$$\bar{d}(Y, \tilde{X}) \leq \bar{d}(X_{\gamma\varsigma}, Y) + \bar{d}(X_{\gamma\varsigma}, \tilde{X}) < 2\varepsilon + r$$

which shows that $Y \in \overline{B_r}(\tilde{X})$.

Theorem 3. 5. If there exists a sequence $Y = (Y_{\gamma\varsigma})$ convergent to $\tilde{X}$ such that $\bar{d}(X_{\gamma\varsigma}, Y_{\eta\gamma\varsigma}) \leq r$ for each $(\gamma,\varsigma) \in \mathbb{N} \times \mathbb{N}$, then a sequence $(X_{\gamma\varsigma})$ is r-statistically convergent to $\tilde{X}$.

Proof. From our assumption, we have

$$\delta_2(\{(\gamma,\varsigma) \in \mathbb{N} \times \mathbb{N}: \bar{d}(Y_{\gamma\varsigma}, \tilde{X}) \geq \varepsilon\}) = 0$$

for every $\varepsilon > 0$. Since $\bar{d}(X_{\gamma\varsigma}, Y_{\gamma\varsigma}) \leq r$ for each (γ,ς), the inclusion

$$\{(\gamma,\varsigma): \bar{d}(Y_{\gamma\varsigma}, \tilde{X}) \geq \varepsilon\} \supseteq \{(\gamma,\varsigma): \bar{d}(X_{\gamma\varsigma}, \tilde{X}) \geq r + \varepsilon\}$$

holds.

Hence, we get

$$\delta_2(\{(\gamma,\varsigma) \in \mathbb{N} \times \mathbb{N}: \bar{d}(X_{\gamma\varsigma}, \tilde{X}) \geq r + \varepsilon\}) = 0,$$

which shows the proof.

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